

Neutrino oscillations and shortcuts in the extra dimension

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Outline

- Large extra dimensions: particle physics & cosmology
- Neutrinos in extra dimensions
- Neutrino oscillations & LSND dilemma
- Bulk shortcuts, neutrino oscillations & LSND
- Neutrino bulk shortcuts:
particle physics, astrophysics & cosmology
- Neutrino wormholes: Tales of the Timeless

Work in Collaboration with S. Pakvasa (Hawaii) and T.J. Weiler (Vanderbilt)

Extra dimensions

- First ideas of extra dimensions: Theodor Kaluza (1921)



and Oskar Klein (1926)

- **Fundamental ingredient of string theory**: consistently formulated in a space-time with 10 or 11 dimensions
- **Traditional picture**: extra dimensions **compactified with radii $\mathcal{O}(l_P) \sim 10^{-33}$ cm**
- **Large extra dimensions allowed, if observable world constrained on a 4-brane**
P. Horava, E. Witten, 1996

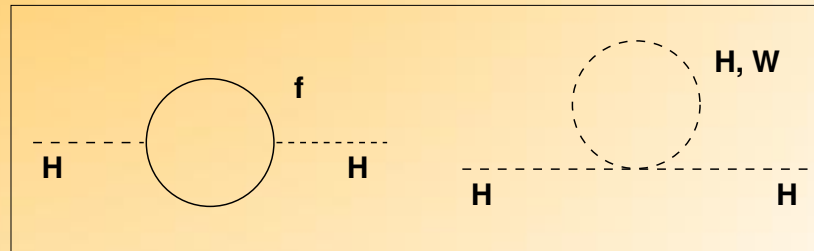
New light
on old problems
in particle physics & cosmology

Hierarchy problem: Why is $m_{EW} \ll M_P$ stable?

Loop corrections to the Higgs mass:

$$\delta m_H^2 = \mathcal{O} \left(\frac{g^2}{16\pi^2} \right) \cdot \int^\Lambda d^4 k \frac{1}{k^2} = \mathcal{O} \left(\frac{\alpha}{\pi} \right) \Lambda^2 \gg m_H^2$$

for cutoff scale $\Lambda \simeq M_P \gg m_{EW} \simeq \text{TeV}$



Large extra dimensions and the hierarchy problem

Large extra dimensions:

$4d$ Planck scale ($M_P = G_N^{-1/2} \sim 10^{19} \text{ GeV}$) \gg fundamental $(4 + \delta)d$
Planck scale M_*

Basic idea:

$$\vec{F}_{\text{EM}} \propto r^{-2} \text{ in } 3d \leftrightarrow \vec{F}_{\text{grav}} \propto r^{-(2+\delta)} \text{ in } (3 + \delta)d$$

generalized Gauss law:

$$M_P^2 = M_*^{\delta+2} V_\delta$$

- V_δ : volume of δ -dimensional extra space
- $M_* \sim \mathcal{O}(m_{EW}) \rightarrow$ large scale M_P completely avoided

N. Arkhani-Hamed, S. Dimoulouos, G.R. Dvali, 1998; I. Antoniadis, N. Arkhani-Hamed, S. Dimoulouos, G.R. Dvali, 1999; L. Randall, R. Sundrum , 1999

Consequences of large extra dimensions

Gravity is increasing strongly at energies $> R^{-1}$

$$R = \sqrt[\delta]{V} / 2\pi \text{ compactification radius}$$

$$M_* \sim 1 \text{ TeV} \rightarrow R \simeq \text{mm-fm for } \delta = 2 - 6$$

Consequences:

- Kaluza-Klein States

- Analogy: particle in a box, momenta of bulk fields are quantized
- 4d perspective: **tower of Kaluza-Klein excitations:**

- $m_n^2 = \frac{n^2}{R^2}.$

- TeV **black hole production at the LHC!**

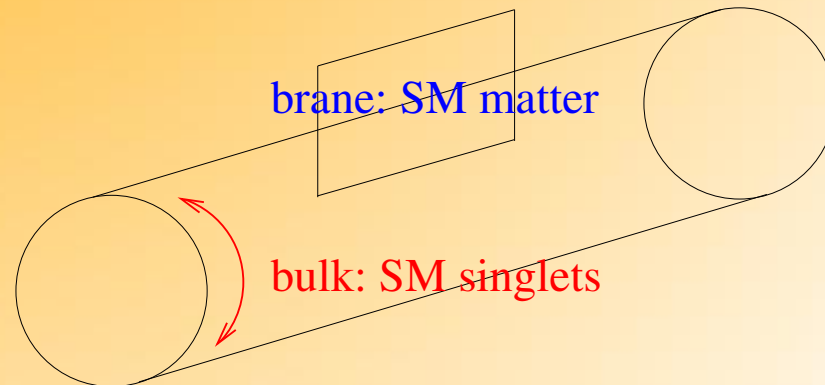
Large extra dimensions and neutrino masses

- **No large scale** → **no seesaw** suppression of neutrino masses
- However: string theories → singlet fermions in the bulk (e.g. superpartners of moduli fields) → ν_R

→ **small Dirac neutrino masses** from **volume-suppressed couplings** to ν_R in the bulk:

$$m^D = \frac{vY}{\sqrt{2V_\delta} M_*^\delta} = v \frac{Y}{\sqrt{2}} \frac{M_*}{M_P}$$

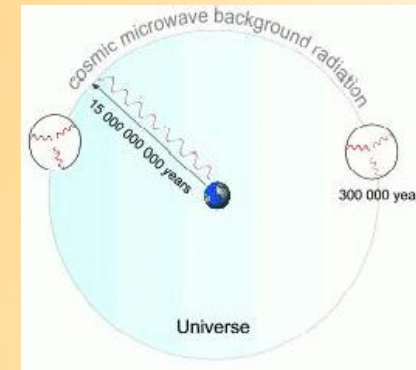
suppression factor: M_*/M_P



N. Arkhoni-Hamed, S. Dimouloulos, G.R. Dvali, J. March-Russel, 1998; K.R. Dienes, E. Dudas, T. Gherghetta, 1999; Y. Grossman, M. Neubert, 2000; S.J. Huber, Q. Shafi, 2002; G. Bhattacharyya, H.V. Klapdor-Kleingrothaus, H. Päs, A. Pilaftsis, 2002

Bulk shortcuts and the horizon problem

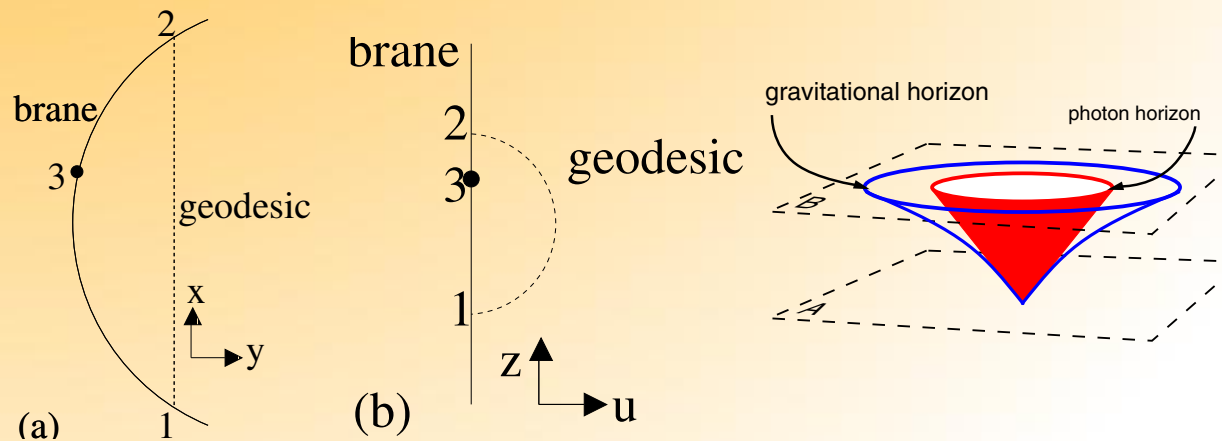
Standard cosmology: Universe homogenous over distances without causal contact (CMB)



- **Conventional solution:** Inflationary epoch in the early universe:

$$R(t) \propto \exp(\sqrt{\Lambda/3}t)$$

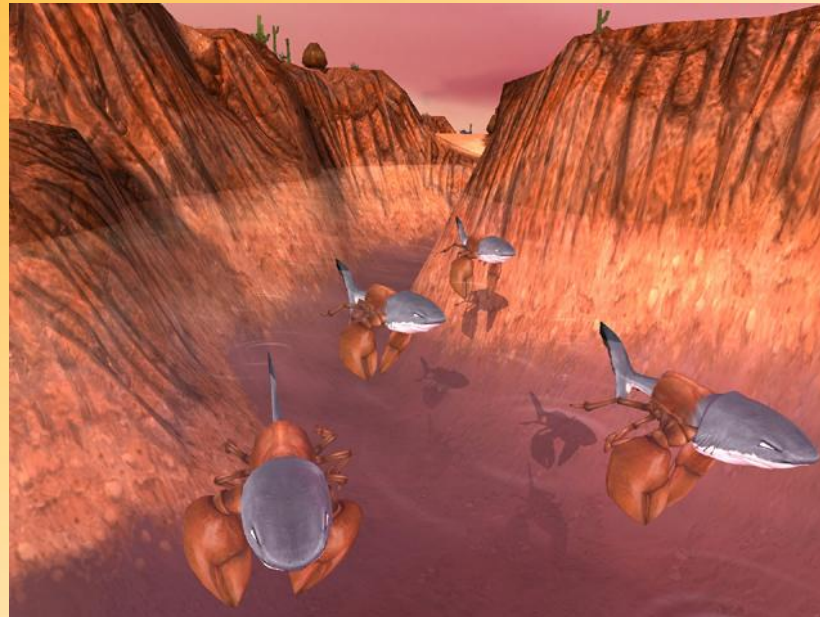
- **Alternative solution:** graviton shortcuts in the extra dimension D.J.H. Chung & K. Freese, 1999; G. Kaelbermann & H. Halevi, 1998; R.R. Caldwell & D. Langlois, 2001



What about
sterile neutrinos

?

SHORtBreak STERileS?

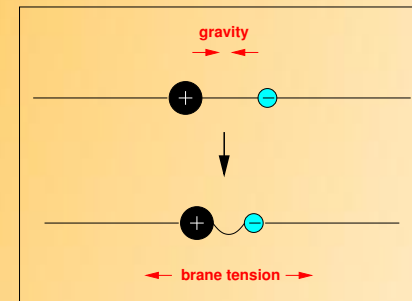


Microsoft: Impossible creatures

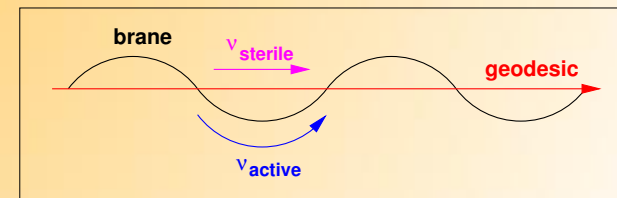
Bulk shortcuts of sterile neutrinos

3 mechanisms for microscopical brane buckles:

- Gravitational attractions between brane bound systems



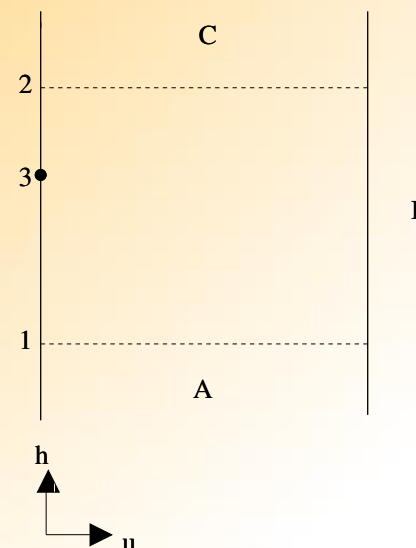
- Quantum fluctuations



- Chung-Freese 2 brane system

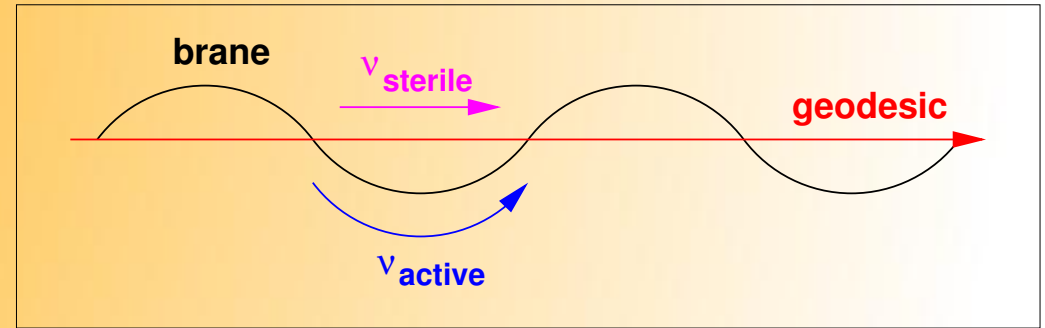
$$ds^2 = dt^2 - [e^{-2ku} a^2(t) dh^2 + du^2]$$

D.J.H. Chung & K. Freese, 1999



Bulk shortcuts of sterile neutrinos

A toy model:



1+2 d Minkowski spacetime: $ds^2 = dt^2 - dx^2 - dy^2$

Periodic brane: $y = \sin kx$

coordinate transformation $(x, y) \rightarrow (u, z)$ straight brane:

$u = y - \sin kx$ and $z = \int \sqrt{1 + k^2 \cos^2 kx} dx$

\rightarrow (uz) -line element: $ds^2 = dt^2 - dz^2 - du^2 - \frac{2 \cos kx(z)}{\sqrt{1+k^2 \cos^2 kx(z)}} du dz$

γ moving along the brane $((u, z)$ system): $z_b = t_f$

ν_s bulk geodesic $((x, y)$ system): $y_g = 0, \quad x_g = t \Rightarrow$

$u_g = -\sin(kt), \quad z_g = \int_0^{t_f} \sqrt{1 + k^2 \cos^2 kt} dt = \frac{\sqrt{(1+k^2)}}{k} \mathcal{E} \left(kt_f, \sqrt{\frac{k^2}{1+k^2}} \right)$

$$z_g > z_b$$

Ratio of travel times: $\epsilon = 1 - \frac{k t_f}{\sqrt{(1+k^2)} \mathcal{E} \left(k t_f, \sqrt{\frac{k^2}{1+k^2}} \right)}$

Toy metric allows for
apparent superluminal
propagation!

How
do bulk shortcuts
affect
neutrino oscillations ?

Neutrino oscillations

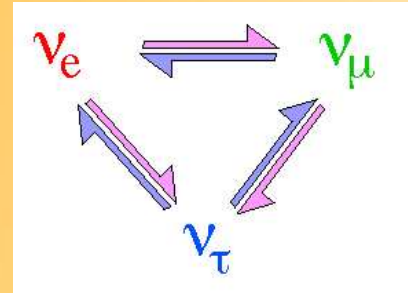


Quantum mechanics: A system is situated in the **coherent superposition of all possible states**

- Cats are simultaneously dead and alive
- **Neutrinos are simultaneously in different eigenstates**

Neutrino oscillations

Solar neutrinos
Atmospheric neutrinos
LSND \Rightarrow



2 conditions:

- Neutrinos possess mass
- Masse eigenstates \neq Flavor eigenstates

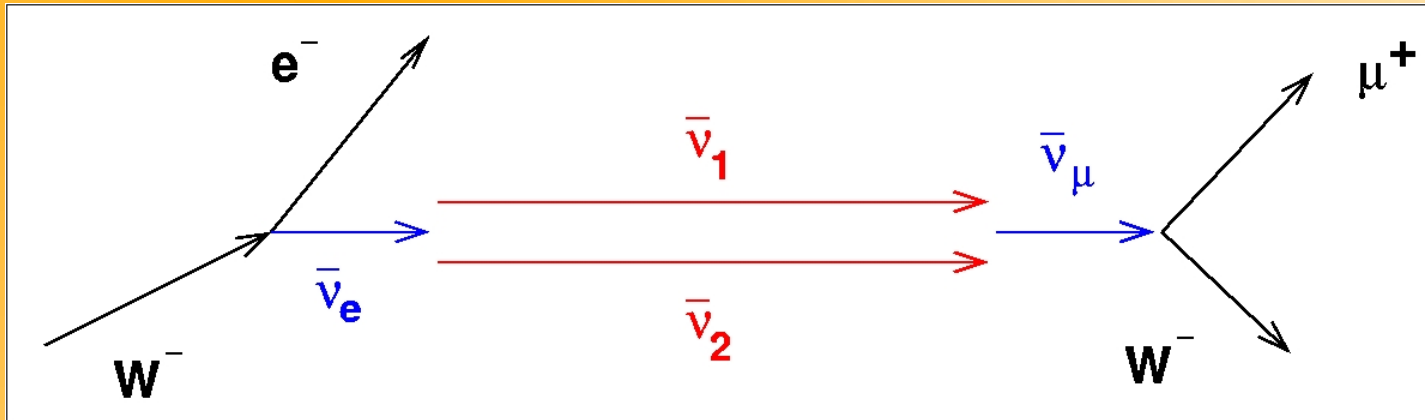
$$|\nu_e\rangle = \sum_{i=1}^3 U_{ei} |\nu_i\rangle$$

2 states system:

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

$$|\nu_e\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle \quad |\nu_\mu\rangle = -\sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle$$

Neutrino oscillations



Defined energy $\Rightarrow p_i = \sqrt{E^2 + m_i^2} \simeq E - \frac{m_i^2}{2E} \quad (m_i \ll E)$

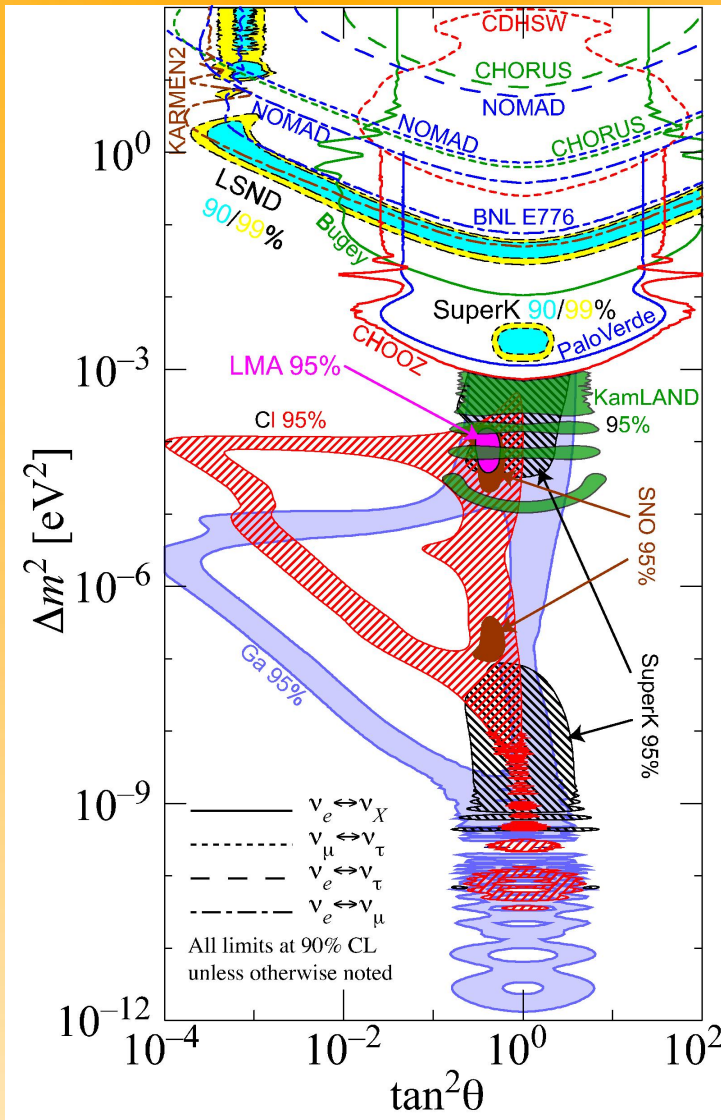
$$|\nu_e(t)\rangle = e^{-iEt + ip_i L} |\nu_e(0)\rangle \quad (t \simeq L)$$

$$|\nu_e(t)\rangle = \sum_i U_{ei} e^{i \frac{m_i^2}{2E} L} |\nu_i\rangle$$

$$P(\nu_e \rightarrow \nu_\mu) = |\langle \nu_\mu | \nu_e \rangle|^2 = \left| \sum_i U_{\mu i}^* e^{i \frac{m_i^2}{2E} L} U_{ei} \right|^2$$

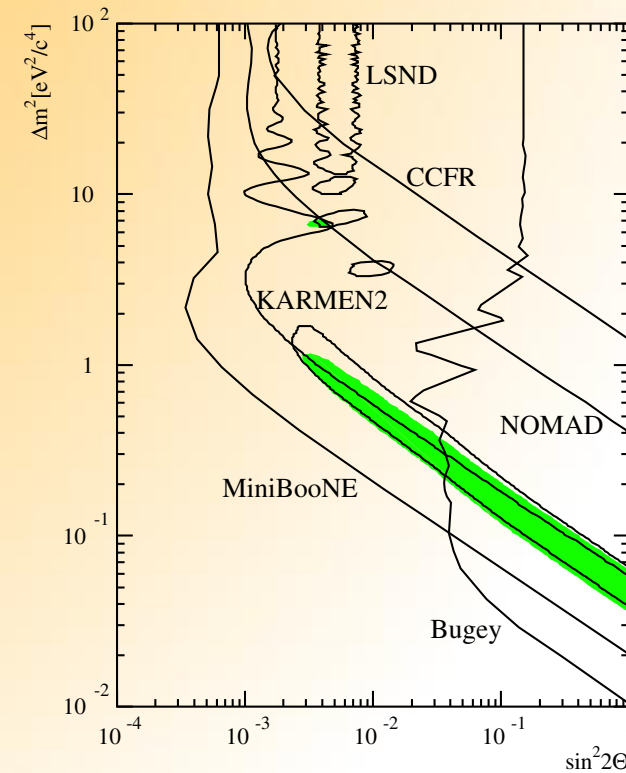
$$P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta) \sin^2\left(\Delta m^2 \frac{L}{4E}\right), \quad \Delta m^2 = m_2^2 - m_1^2$$

The LSND Dilemma

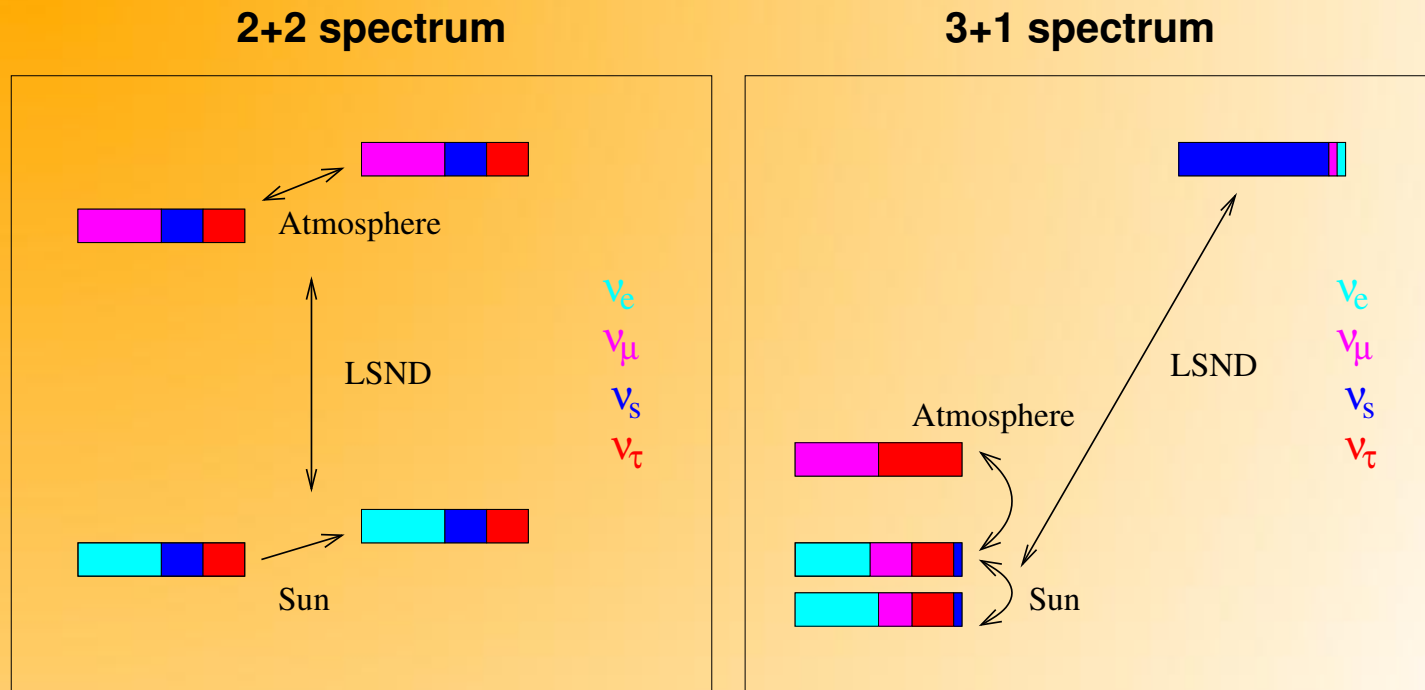


- 3 Δm^2 's \rightarrow 4 neutrinos!

- width of the Z-boson (LEP) \rightarrow 3 neutrinos!
- \rightarrow one **sterile neutrino?** (i.e. not coupling to the Z)



LSND and sterile Neutrinos



2+2 spectrum:

no oscillations of solar or atmospheric ν 's into steriles!

3+1 spectrum:

BUGEY bound: $\sin^2 2\theta_{e\phi} = 4U_{e4}^2 (1 - U_{e4}^2)$

CDHS bound: $\sin^2 2\theta_{\mu\mu} = 4U_{\mu4}^2 (1 - U_{\mu4}^2)$

LSND: $\sin^2 2\theta_{\text{LSND}} = 4U_{e4}^2 U_{\mu4}^2$

LSND is doubly suppressed! $\sin^2 2\theta_{\text{LSND}} \simeq \frac{1}{4} \sin^2 2\theta_{e\phi} \sin^2 2\theta_{\mu\mu}$

Bulk shortcuts and neutrino oscillations

Evolution equation in flavor space:

$$i \frac{d}{dt} \begin{pmatrix} \nu_a(t) \\ \nu_s(t) \end{pmatrix} = H_F \begin{pmatrix} \nu_a(t) \\ \nu_s(t) \end{pmatrix}$$

Hamiltonian in the presence of bulk shortcuts:

$$H_F = + \frac{\delta m^2}{4E} \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta \end{pmatrix} + E \frac{\epsilon}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

⇒ A Resonance exists at $E_{\text{res}} = \sqrt{\frac{\delta m^2 \cos 2\theta}{2\epsilon}}$

→ choose $E_{\text{res}} = 60 - 500 \text{ MeV} \leftrightarrow \epsilon \simeq 10^{-18} - 10^{-16}$

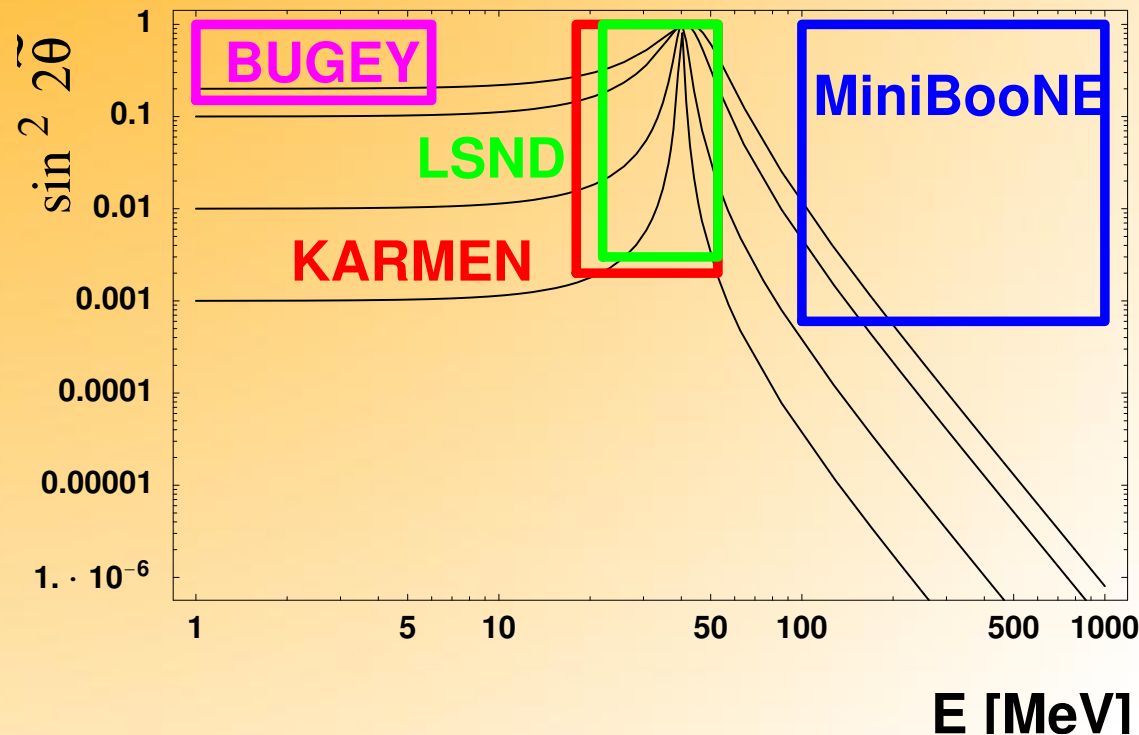
The active-sterile oscillation probability

$$P_{as} = \sin^2 2\tilde{\theta} \sin^2(\delta H D/2)$$

$$\sin^2 2\tilde{\theta} = \left[\frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \right]$$

$$\delta H = \frac{\delta m^2}{2E} \sqrt{(\cos 2\theta - A)^2 + \sin^2 2\theta}$$

$$A = (E_{\text{res}}/E)^2$$



Oscillations at $E \gg E_{\text{res}}$ are suppressed!

LSND in the bulk shortcut scenario

Active-sterile neutrino mixing:

$$|\nu_a\rangle = \cos\theta_* |\nu_\mu\rangle + \sin\theta_* |\nu_e\rangle$$

BUGEY bound ($E_\nu \simeq$ (few) MeV $\ll E_{\text{res}}$):

$$\sin^2 2\theta_{e\mu} \simeq 4 \sin^2\theta \sin^2\theta_*$$

LSND & KARMEN ($E_\nu = 20 - 58.82$ MeV):

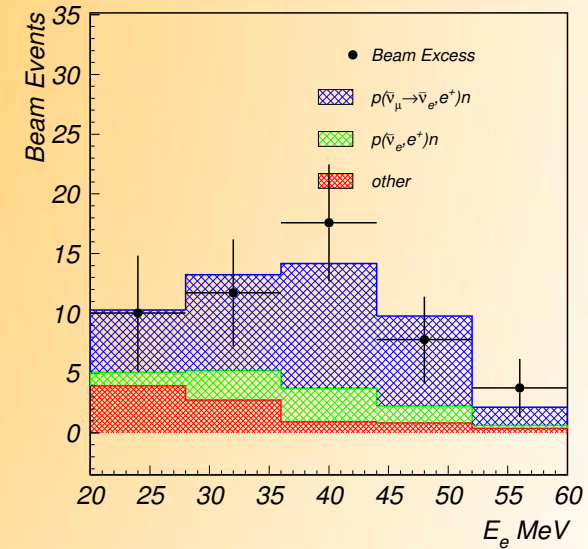
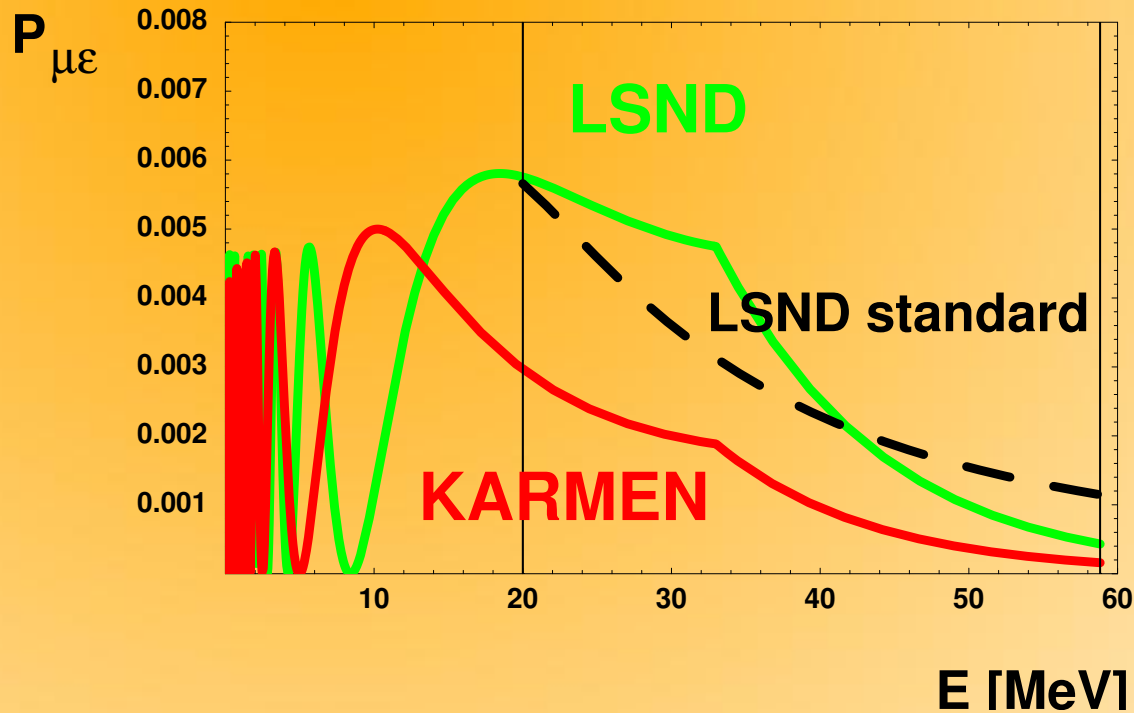
$$\sin^2 2\tilde{\theta}_{\text{LSND}} = \frac{1}{4} \sin^2 2\theta_* (1 \mp \cos 2\tilde{\theta})^2$$

CDHS bound ($E_\nu > 1$ GeV $\gg E_{\text{res}}$):

$$\sin^2 \tilde{\theta}_{\mu\mu} \simeq \cos^2\theta_* \sin^2 2\theta \left(\frac{E}{E_{\text{res}}}\right)^{-4}$$

The CDHS bound is suppressed by its large energy!

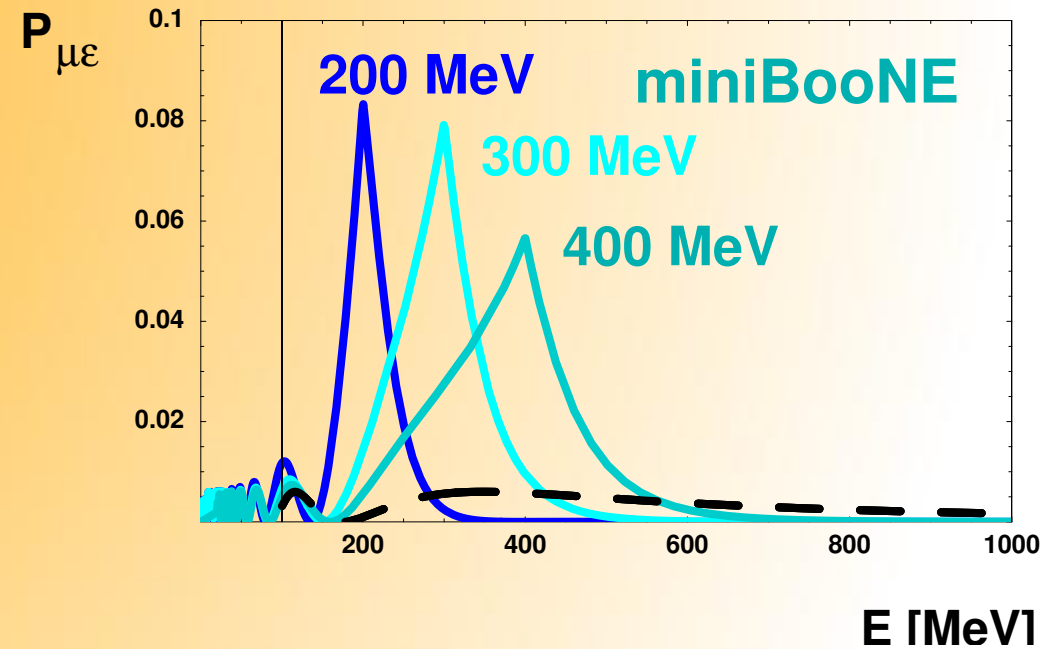
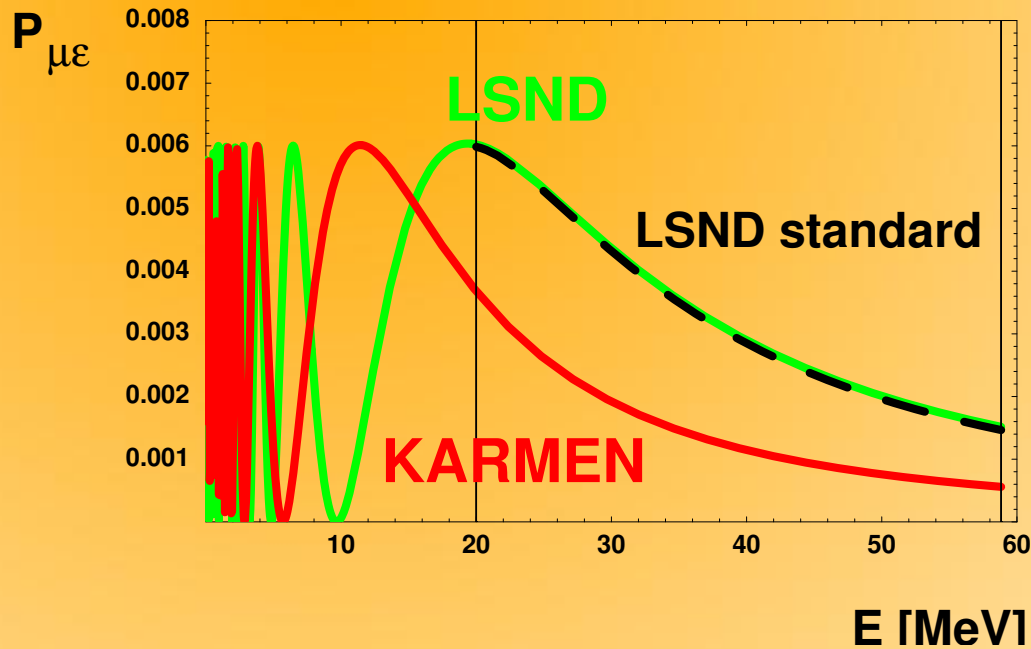
Scenario with low resonance energy



- $E_{\text{res}} = 33 \text{ MeV}; \sin^2 \theta_* = 0.01;$
 $\sin^2 2\theta = 0.9; \delta m^2 = 0.7 \text{ eV}^2$
- $P_{\text{LSND}} > P_{\text{KARMEN}}$

- good (better) fit of LSND spectrum
- no signal at miniBooNE!

Scenario with high resonance energy



- $E_{\text{res}} = 200 \text{ MeV}, 300 \text{ MeV}, 400 \text{ MeV}; \sin^2 \theta_* = 0.1; \sin^2 2\theta = 0.45;$
 $\delta m^2 = 0.8 \text{ eV}^2$
- good fit to LSND spectrum, $P_{\text{LSND}} > P_{\text{KARMEN}}$
- enhanced miniBooNE signal in the energy range 100-600 MeV

Big Bang Nucleosynthesis

Prediction of primordial abundances of light elements: major success of Big Bang Cosmology

Problem with sterile neutrinos:

ν oscillations populate extra species in early universe:

$$\rho_{\nu_s} = \frac{7}{8} \rho_\gamma$$

- \rightarrow faster expansion of the universe
- \rightarrow higher temperature for weak freezeout
- \rightarrow more neutrons \rightarrow larger ${}^4\text{He}$ abundance

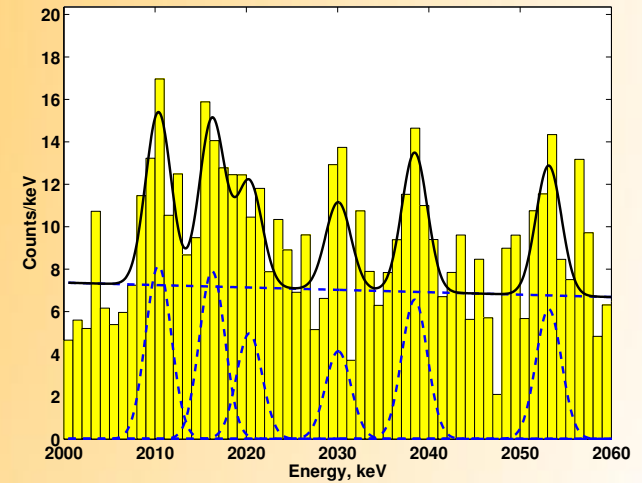
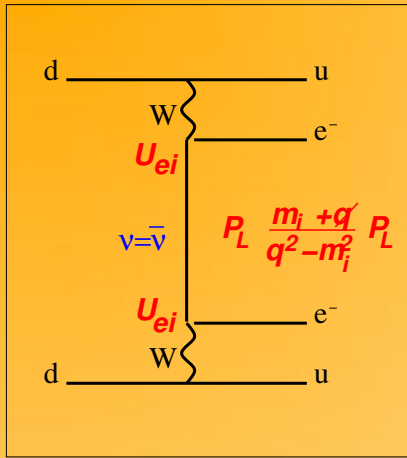
Bulk shortcut scenario:

- higher density: larger brane buckle effect due to gravitational attraction
- higher temperature: more quantum fluctuations
- higher density: more scattering off the brane in Chung-Freese scenario

All cases: larger $\epsilon \rightarrow$ smaller E_{res}

If $E_{\text{res}} \lesssim 3 \text{ MeV}$: oscillations suppressed

Neutrinoless double beta decay



$$\Gamma_{0\nu\beta\beta} \propto |m_{ee}|^2 = |\tilde{U}_{e4}^2 \tilde{m}_4 + \sum_{j=1,3} \tilde{U}_{ej}^2 m_j|^2$$

$m_{ee} = 0.1 - 0.9$ eV? (Klapdor-Kleingrothaus, Krivosheina, Dietz, Chkvorets 2004)

- $p_\nu \sim p_F \sim \mathcal{O}(100)$ MeV
- $p_\nu \sim E_{\text{res}} \Rightarrow \tilde{U}_{e4}$ may become large!
- large signals for $0\nu\beta\beta$ decay!

Tritium β decay ($E_\nu \simeq \mathcal{O}(\text{MeV})$): no enhancement

Supernova ν 's

- r-process nucleosynthesis:

rapid capture of neutrons on iron-sized seed nuclei:
prime candidate for the synthesis of nuclei heavier than iron

- prevented from $\nu_e n \rightarrow pe \rightarrow \alpha$ particles
- $\nu_e n \rightarrow pe$ can be suppressed sufficiently by strong $\nu_e \rightarrow \nu_s$ oscillations
G.M. Fuller, 1999
- Bulk shortcut effects: the matter resonant conversion will be cut off above E_{res}

Dark Matter & Horizon Problem

LSND neutrino & keV warm dark matter

WDM solves:

- cuspy core problem of cold dark matter
[S.Dodelson, L. Widrow, 1993](#); [X.d. Shi, G.M. Fuller, 1998](#)
- induce observed velocities of radio pulsars
[G.M. Fuller, A. Kusenko, I. Mocioiu, S. Pascoli, 2003](#)

$$\delta m^2 \rightarrow \delta m^2 \sqrt{(\cos 2\theta - A)^2 + \sin^2 2\theta} \ll \delta m^2?$$

However: small $\sin^2 2\theta$ seems not to fit LSND spectrum

Sterile neutrinos & the horizon problem

Are sterile neutrino shortcuts superior to graviton shortcuts?

- Bounds from precision experiments on gravitational square law do not apply
- Sterile neutrinos may couple more strongly (Homogeneity problem)

If
sterile neutrinos
propagate with superluminal velocity...
possibility of time travel?

Can sterile neutrino shortcuts be interpreted as
Wormholes?

*“Psycho-ceramics warning: Crackpots are politely requested to refrain from reading
this paragraph”*

(Matt Visser, Lorentzian Wormholes)

sorry, Matt....

Wormholes and time machines

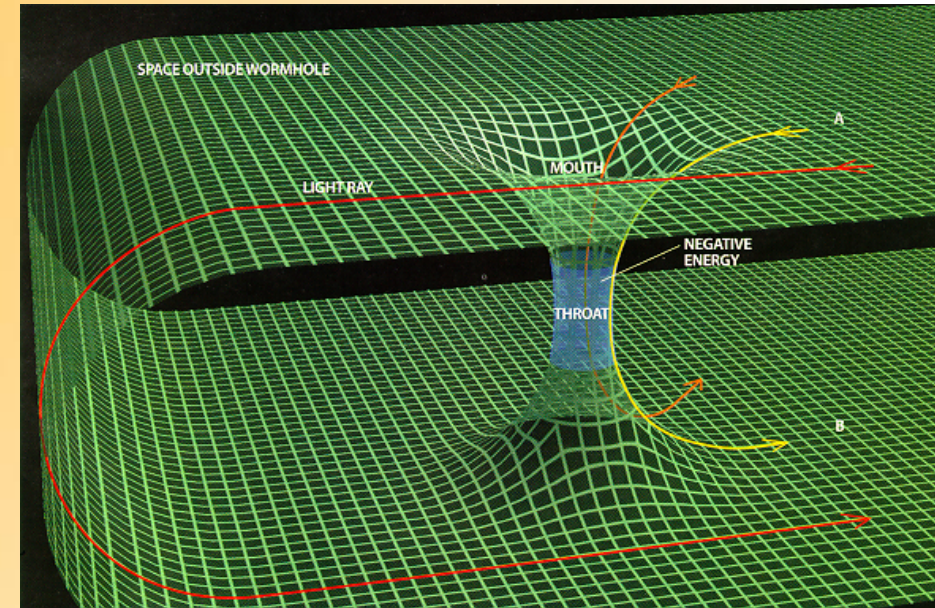
Matt Visser's 3 steps

1. Find a wormhole
(*"recipe for dragon stew"*)

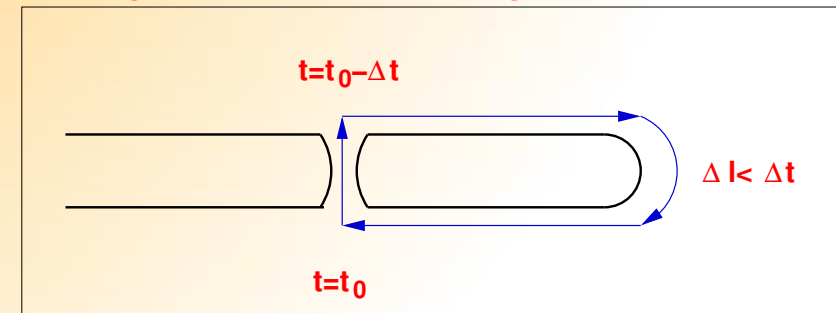


T. Roman, Scientific American

2. Introduce a time-shift



3. Bring the mouths together



Morris, Thorne, Yurtserver, PRL 61 (1988) 1446

Time machine proposals

- Gödels rotating universe (1949)
- Van Stockums and Tiplers rotating cylinders (1937 & 1974)
- Gott's pair of moving cosmic strings (1991)
- Wheelers space time foam (1962)
- Kerr and Newman geometries (1973)
- Alcubierres warp drive (1994)

Obstacles:

- exotic energy violating the energy conditions → strong curvature
- unphysically fast rotation → tipping Lorentz cones

Timekeepers



“The Time Keepers were born at the end of time, entrusted with the safety of Time by He Who Remains. The Time Keepers were meant to watch over the space time continuum just outside of Limbo, and make sure the universe thrived.” Marvel Comics

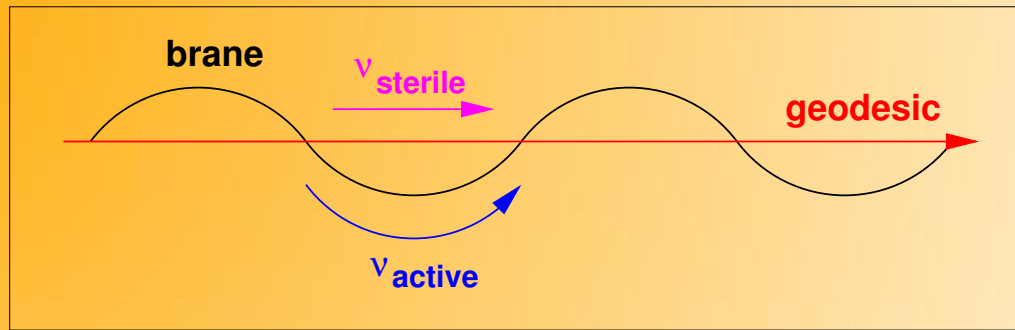
“It seems that there is a chronology protection agency which prevents the appearance of closed time-like curves and so makes the universe safe for historians.”

S.W. Hawking, The chronology protection conjecture, Phys. Rev. D 46 (1992) 603-611



Lenny Susskind: *“Wormholes and time travel? Not likely.”* (gr-qc/0503097)

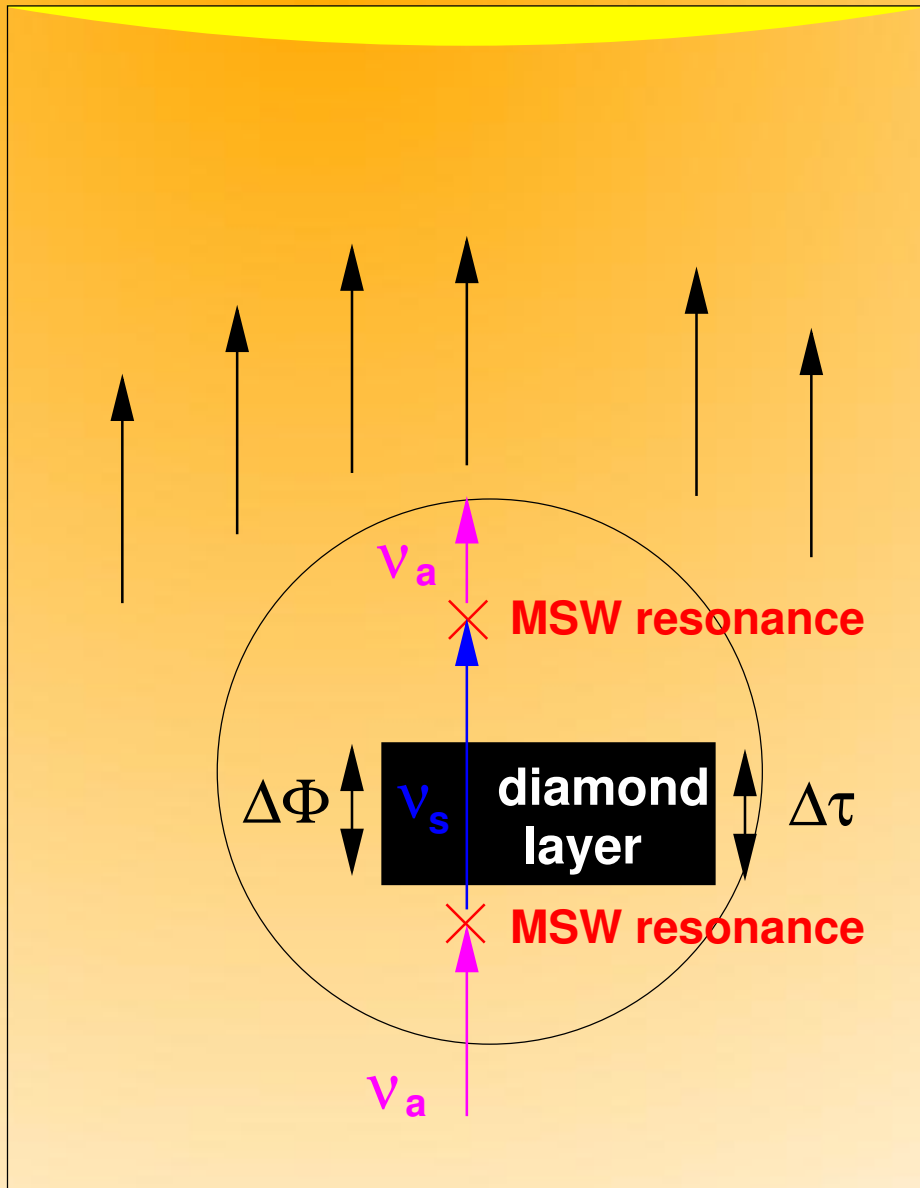
Beating the Timekeepers?



Sterile neutrino shortcuts
allow wormhole-like paths
to points with space-like distance
without rotation or strong curvature!

Sterile neutrino physics may allow
and superluminal communication or back-in-time travel
or studies of physics near the chronology horizon!

Neutrino time machine



M. Kuchner, S. Seager, Aspen 2005

$\Delta\tau$ can be of order seconds!

Conclusions

- Bulk shortcuts may arise in extra dimensional theories
- Bulk shortcuts affect neutrino mixing and imply a new resonance
- Neutrino oscillations are suppressed for $E \gg E_{\text{res}}$
- LSND becomes compatible with BUGEY and CDHS ($E \gg E_{\text{res}}$)
- $E_{\text{res}} < 100$ MeV: no signal at miniBooNE
- $E_{\text{res}} \gg 100$ MeV: enhanced oscillations at miniBooNE
- BBN bound can be evaded
- Neutrinoless double beta decay may be enhanced
- Supernova ν 's: E_{res} cutoff for MSW effect
- Dark matter, horizon problem, time travel...