Probing the Neutrino Mass Hierarchy in INO using the Atmospheric ν_μ Survival Rate

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Outline . . .

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- Importance of the Mass Hierarchy and future prospects of its detection

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- Results:Detecting the Mass Hierarchy in atmospheric ν_μ events
- Conclusions

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- Over the past few years, goals in neutrino physics have been redefined and increasingly focussed. The present goal is to determine, as precisely as we can, the elements of the leptonic mixing matrix and the masses/mass-squared differences of neutrinos.
- Precision allows us to identify or exclude special valued mixing angles like $\theta_{13} = 0^o, \theta_{23} = 45^o$, and special relations between the quark and lepton sectors like $\theta_{12} + \theta_C = 45^o$, check for unitarity of 3 generations, non-standard interactions, decoherence scenarios.......

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- A multi-pronged effort to acheive these goals is underway via various operating, planned and proposed experiments, *e.g.* long-baseline, reactor, atmospheric, solar, beta-decay, ν-less double beta decay, precision cosmology etc.

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- The 3×3 neutrino mixing matrix U in the MNS parametrization is:

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ ($\theta_{ij} =$ mixing angle between i^{th} & j^{th} mass states). $\delta =$ CP phase.

Introduction . . .

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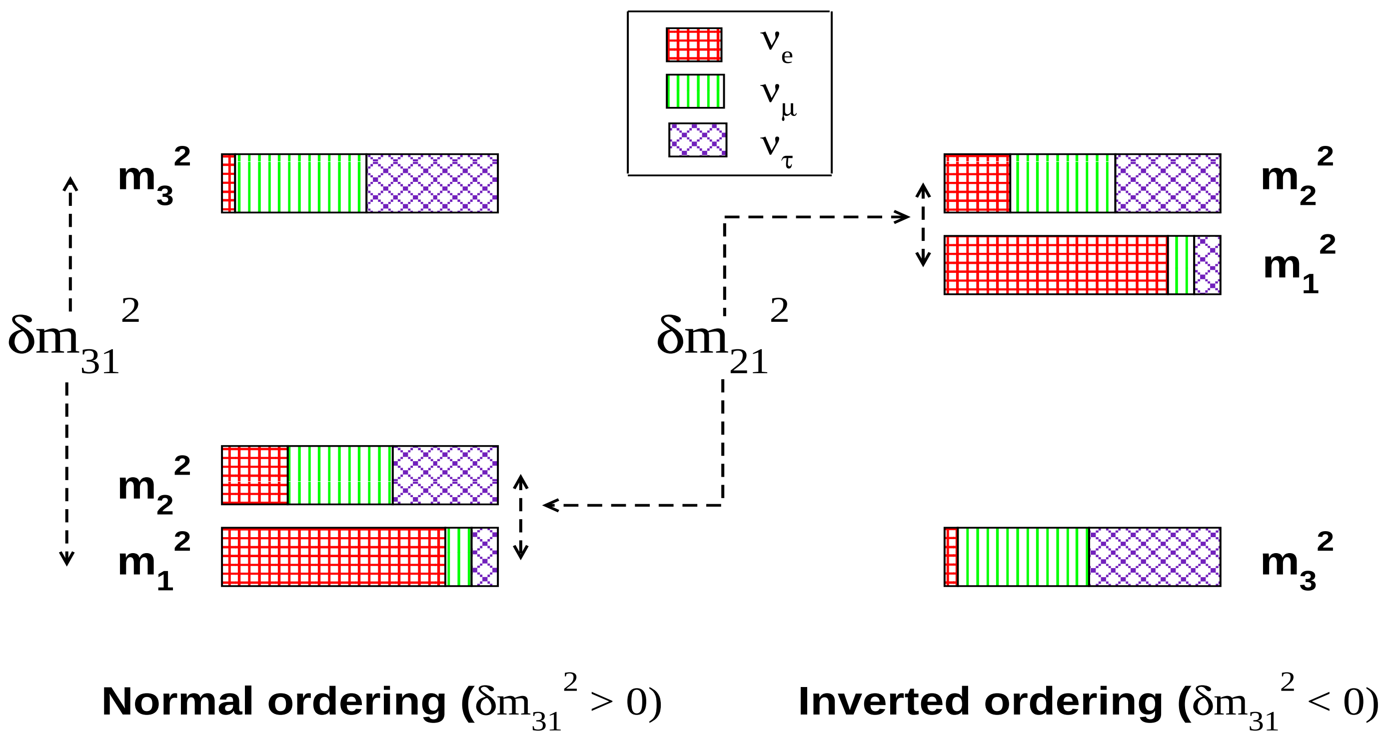
  - $\tan^2\theta_{12} = 0.39^{+0.05}_{-0.04}$ solar,reactor
  - $\tan^2\theta_{23} = 1.0^{+0.3}_{-0.3}$ atmospheric, K2K
  - $\theta_{13} < 12^o @ 3\sigma$ reactor, atmospheric
  - $\delta m^2_{21} = 8.2^{+0.3}_{-0.3} \times 10^{-5} eV^2$ solar,reactor
  - $|\delta m^2_{31}| = 2.2^{+0.6}_{-0.4} \times 10^{-3} eV^2$ atmospheric, K2K
  - $m_\beta < 2.2$ eV beta decay
  - $m_{\beta\beta0\nu} < 0.3$ eV ν less double beta decay
  - $\Sigma m_i < 1.6 - 0.7$ eV Precision Cosmology
  - δ_{CP} is unknown

ν *The Mass Hierarchy, its Significance and Detection*

So far we only know $|\Delta m^2_{31}|$ and not its Sign



The Mass Hierarchy . . .

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- A large class of GUTS use the Type I seesaw mechanism to unify quarks and leptons. Several positive features are lost if in such models the neutrino hierarchy is *inverted* rather than *normal*
  C. Albright, Phys. Lett. **B599**, 285 (2004)

The Mass Hierarchy . . .

- An *inverted* hierarchy on the other hand would generally require m_1, m_2 to be *degenerate*, hinting towards an *additonal global symmetry in the lepton sector.*

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- The type of hierarchy impacts the effectiveness of *leptogenesis* in most theoretical models.

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  ➢ Nu Factories substantial improvement in bounds

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- Detection of the hierarchy via the $\nu_\mu \rightarrow \nu_e$ channel, at not-so-long baselines, is hampered by a $\delta_{CP} - sign(\Delta_{31})$ degeneracy which reduces sensitivity. Additionally, in “off-resonance” sitiuations, there is a $\theta_{13} - sign(\Delta_{31})$ that must be taken into account.

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- Overcoming this in superbeam experiments would require using either *multiple oscillation channels, 2 baselines, different energies, the magic baseline, 2 off-axis locations* or a combination of these stategies.

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- These degeneracies are absent in methods which utilise the muon survival probability.

Matter effects in $P_{\mu e}$

- Vacuum 1-MSD limit ($\Delta m^2_{21} = 0$) :

$$\mathcal{P}_{\mu e} = \sin^2\theta_{23}\sin^2(2\theta_{13})\sin^2\left[\Delta_{31}L/E\right]$$

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- and $\Delta^m_{31} = \Delta_{31}\sqrt{(\frac{A}{\delta m^2_{31}} - \cos 2\theta_{13})^2 + \sin^2 2\theta_{13}}$

Matter effects in $P_{\mu e}$

- We note that: $\mathcal{P}^m_{\mu e}$ is maximized not just at resonance but when the combination $\sin^2(2\theta^m_{13})\sin^2[\Delta^m_{31} L/E]$ is maximal, *i.e*i.e. when $E = E_{res} = E^m_{peak}$.

Matter effects in $P_{\mu e}$

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- For Earth baselines, this translates to:

$$[\rho L]_{\mu e}^{max} = \frac{(2p+1)\pi \times 5.18 \times 10^3}{\tan 2\theta_{13}} \ km\ gm/cc$$

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- *Co-relations and degeneracies play an important role and reduce sensitivity when $\mathcal{P}^m_{\mu e}$ is used to determine the hierarchy*

Matter effect on $P(\nu_\mu \to \nu_\tau)$

- The 1-MSD vacuum expression is:

$$\mathcal{P}^{v}_{\nu_\mu \to \nu_\tau} = \cos^2\theta_{13}\sin^2 2\theta_{23}\sin^2(\Delta_{31})$$

$$- \cos^2\theta_{23} P^{vac}_{\nu_\mu \to \nu_e}$$

- The 1-MSD analytic expression in matter is:

$$\mathcal{P}^{m}_{\nu_\mu \to \nu_\tau} = \cos^2\theta^m_{13}\sin^2 2\theta_{23}\sin^2[(\Delta_{31} + A + \Delta^m_{31})/2]$$

$$+ \sin^2\theta^m_{13}\sin^2 2\theta_{23}\sin^2[(\Delta_{31} + A - \Delta^m_{31})/2]$$

$$- \cos^2\theta_{23} P^{mat}_{\nu_\mu \to \nu_e}$$

→

Maximizing the matter effect in $\mathcal{P}_{\mu\tau}$

- $E = E_{res} = E^{v}_{peak}$ leads to:

$$\Delta\mathcal{P}_{\mu\tau} \simeq \cos^4[\sin 2\theta_{13}(2p+1)\pi/4] - 1$$

(with $\sin^2 2\theta_{23} = 1$, $\cos^2\theta_{13}, \cos 2\theta_{13} \simeq 1$).

- The maximum matter effect condition for L is:

$$[\rho L]^{max}_{\mu\tau} = (2p+1)\pi \times 5.18 \times 10^3 \times \cos 2\theta_{13} \;\; km \; gm/cc$$

The maximum matter effect occurs for $L = 9700$ km for p=1 & $\sin^2 2\theta_{13} = 0.1$ (9300 km, 9900 km for 0.2, 0.05).

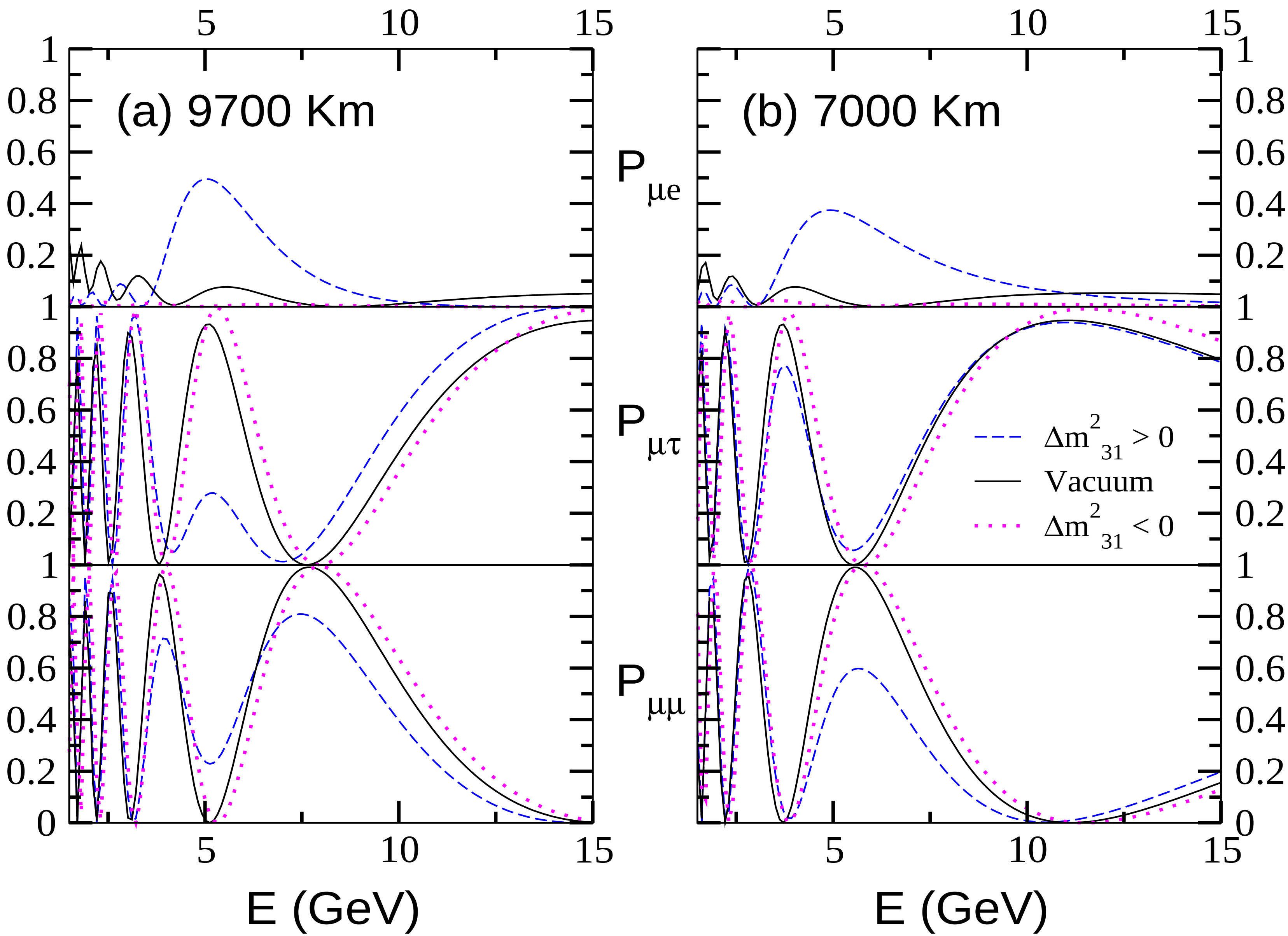
Summary of Conditions for Large Matter effects . . .

Thus, conditions for maximizing the matter effects at very long baselines are:

- $[\rho L]^{\max}_{\mu e} = \frac{(2p+1)\pi 5.18\times 10^3}{\tan 2\theta_{13}}$ Km gm/cc.
- $[\rho L]^{\max}_{\mu\tau} = (2p+1)\pi 5.18 \times 10^3 \cos 2\theta_{13}$ Km gm/cc.
- $[\rho L]^{\max}_{\mu\mu} = p\pi \times 10^4 \cos 2\theta_{13}$ Km gm/cc.

Plot of Probabilities . . .

R. Gandhi *et al.*, Phys. Rev. Lett. **94**, 051801 (2005); hep-ph/0411252



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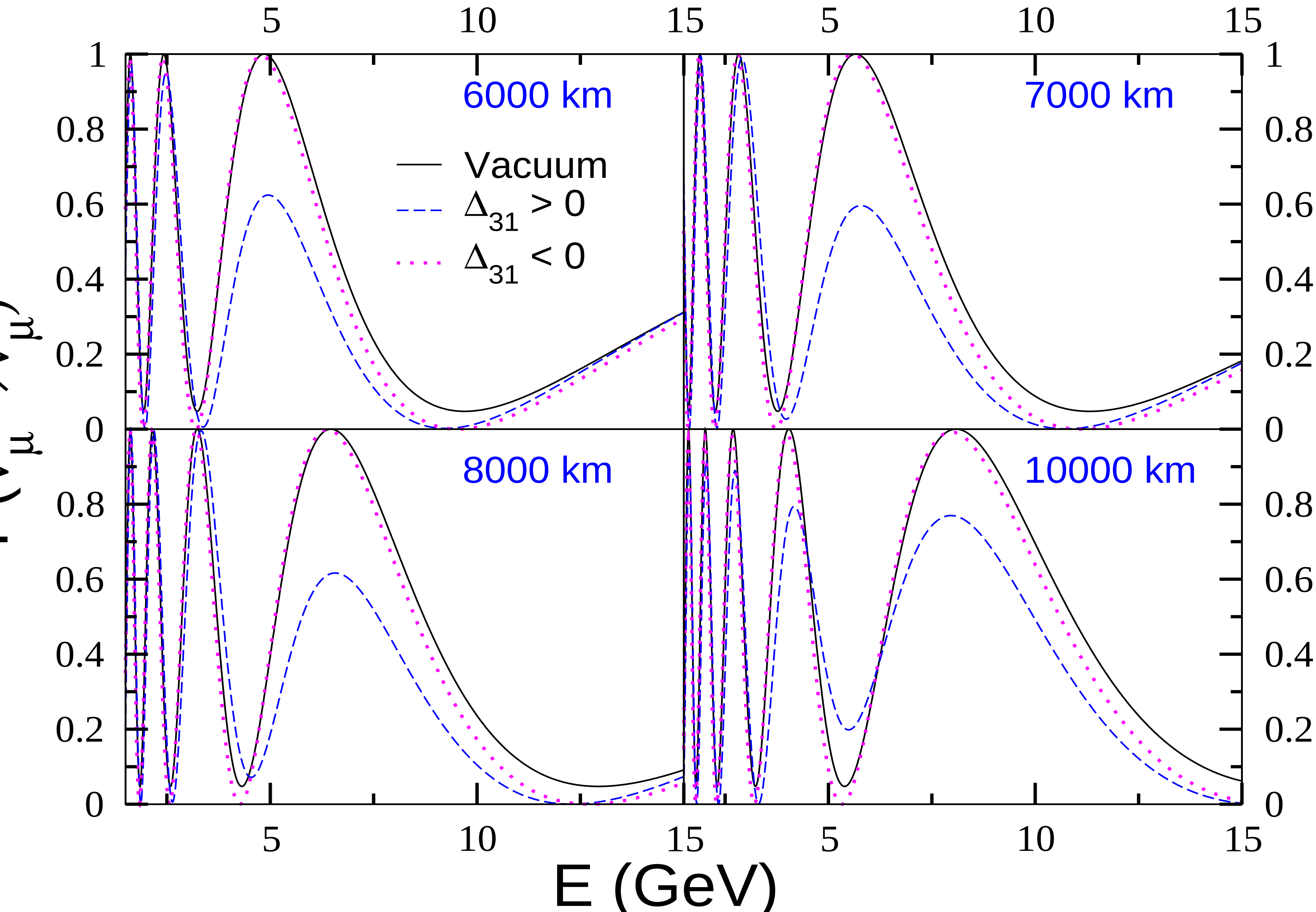
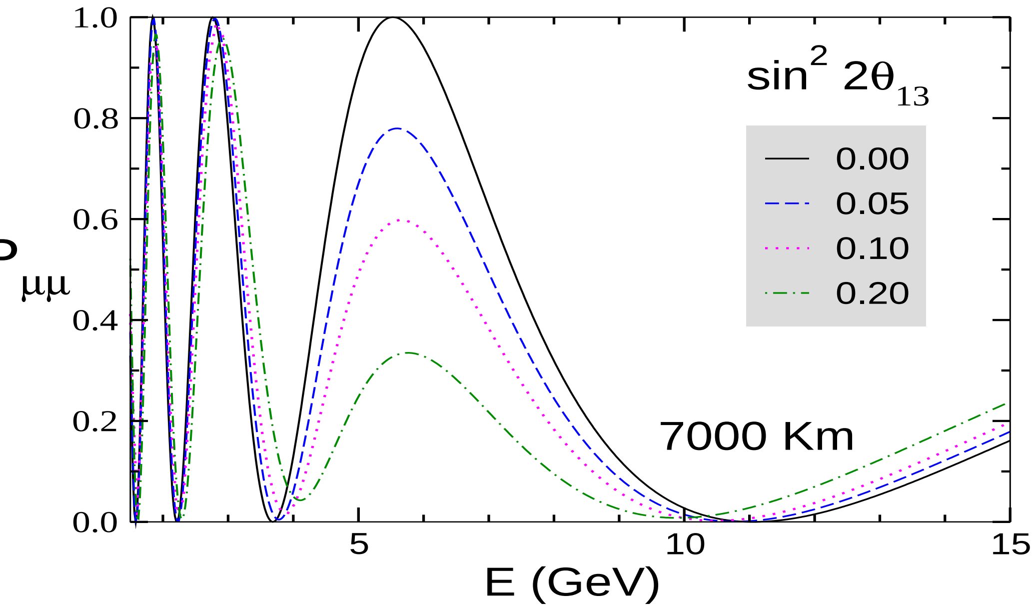


Survival Probability : θ_{13} sensitivity . . .



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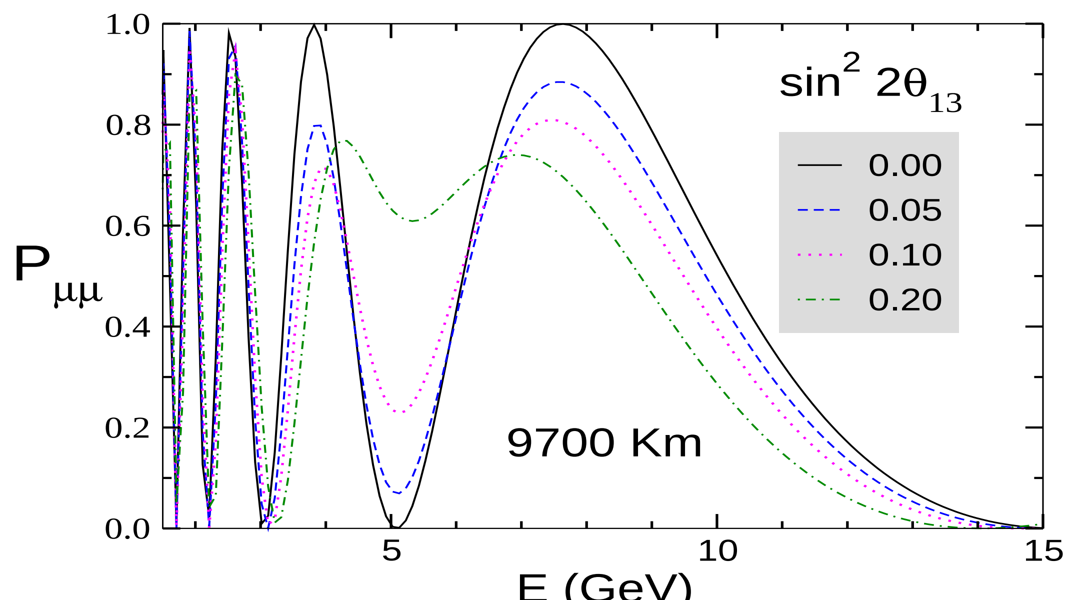
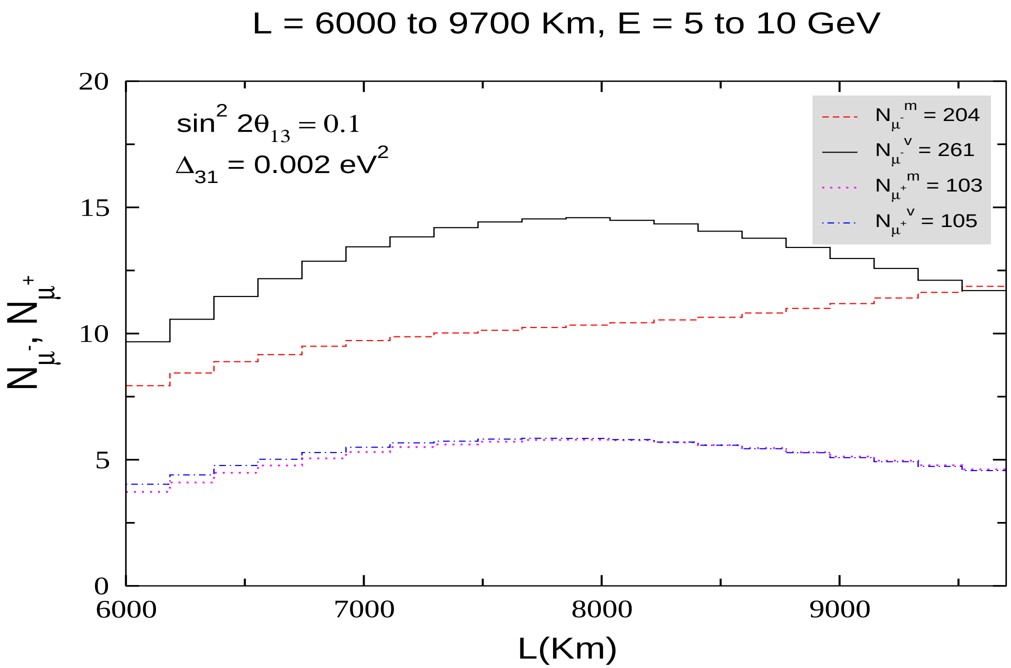
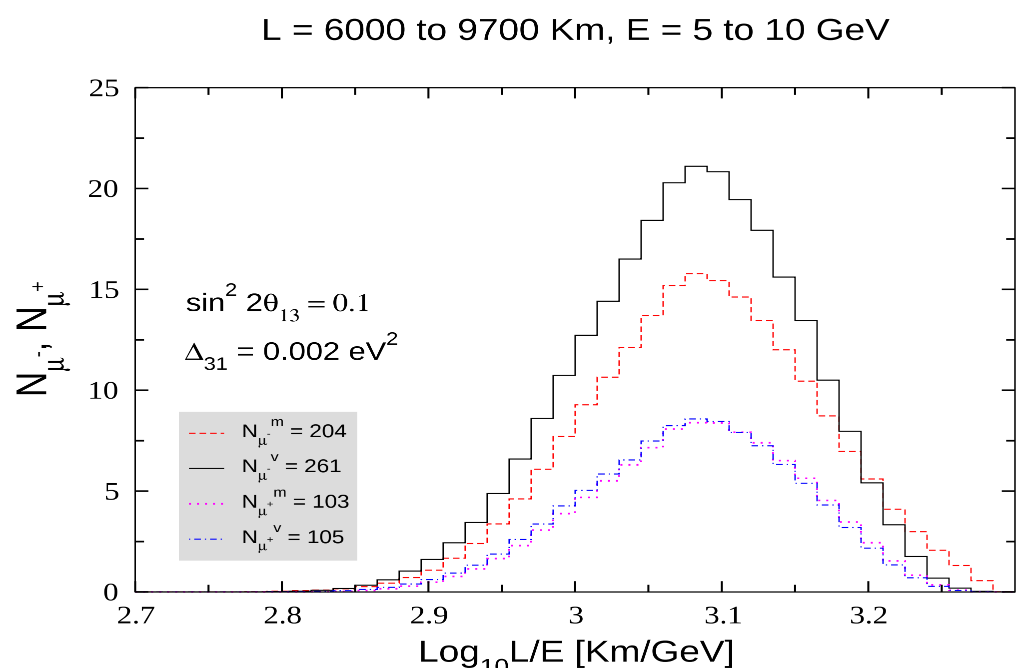


Results for an Iron Calorimeter type detector

Results : Iron Calorimeter, 1000 kt-yr . . .





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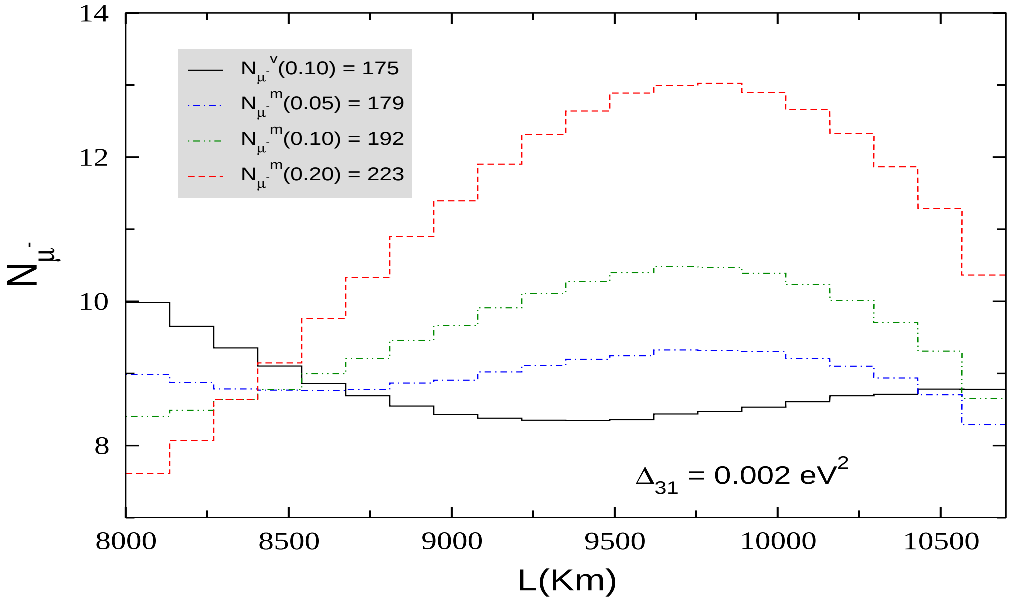
L = 8000 to 10700 Km, E = 4 to 8 GeV



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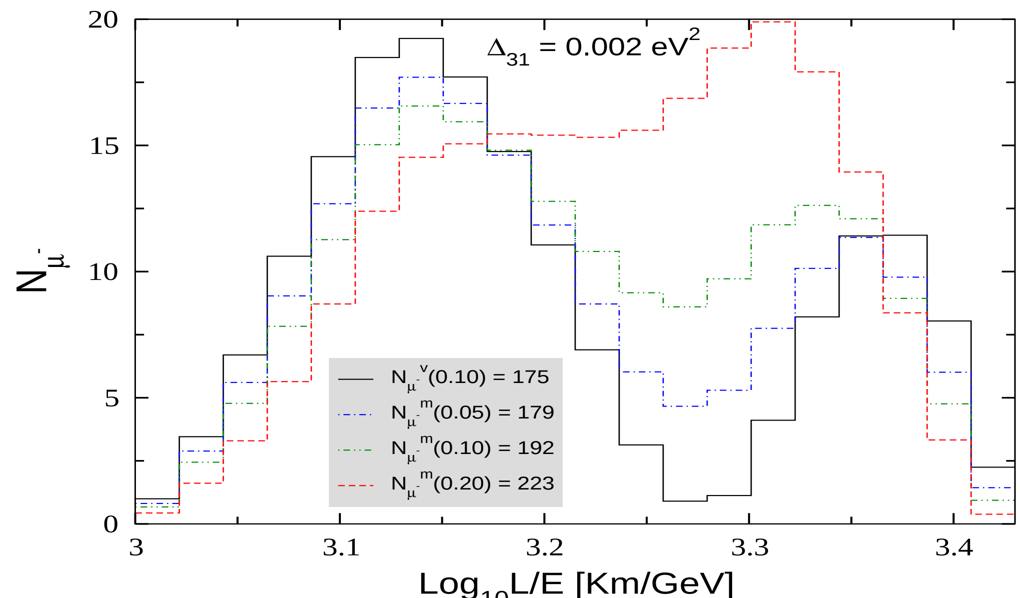

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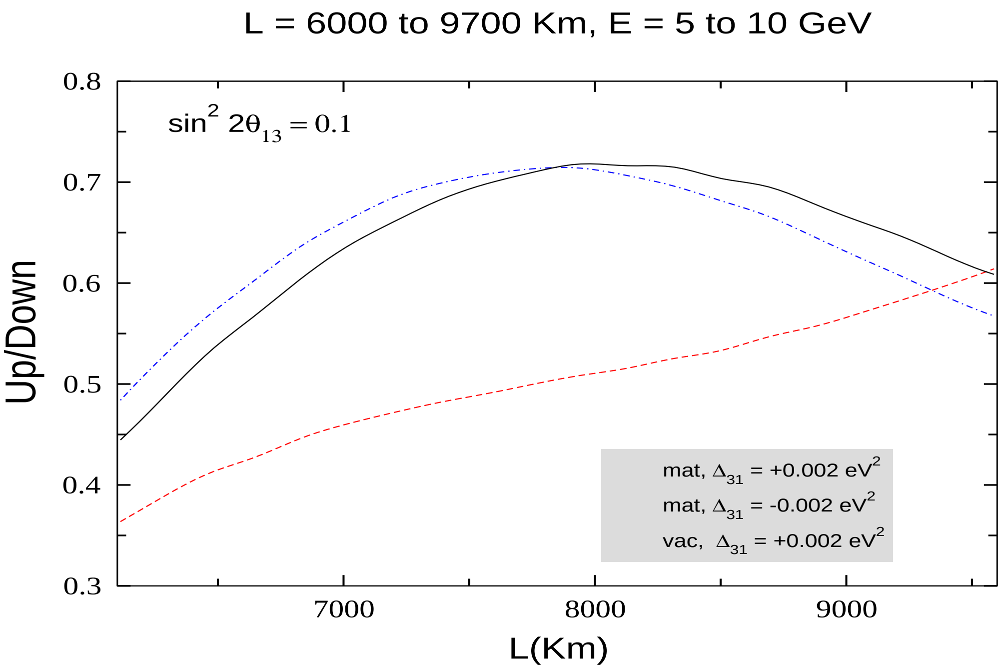

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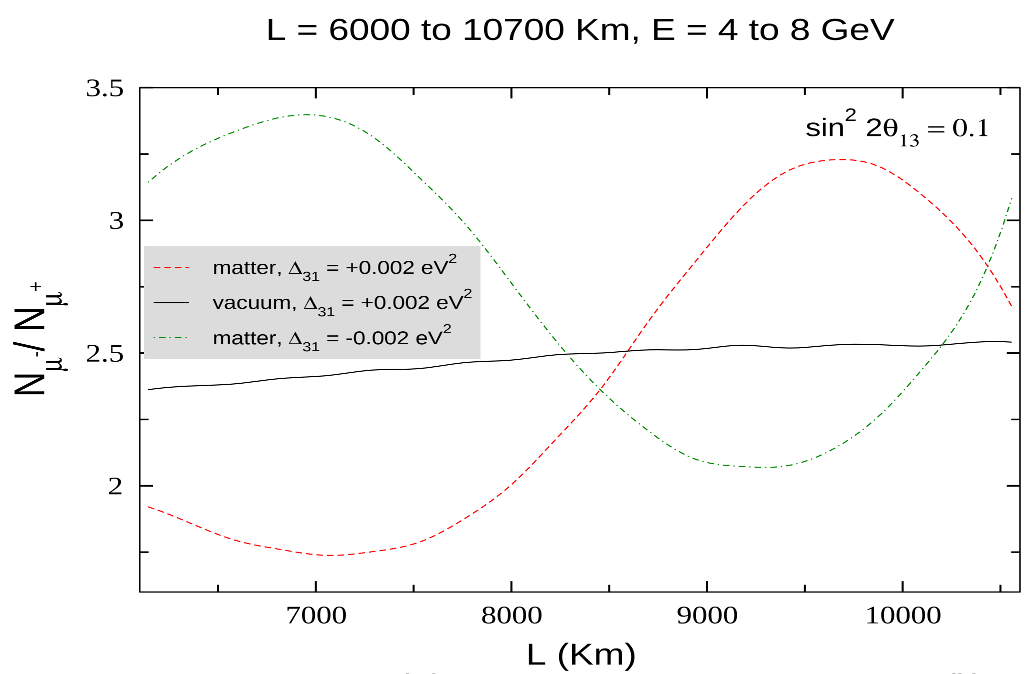

Fe Calorimeter, 1000kt-yr, $\Delta\chi^2$ and σ sensitivity . . .

Choice of the optimal ranges for μ^- events for 1000 kiloton-yr:

Range 1 : E = 5 - 10 GeV and L = 6000 - 9700 Km

$\sin^2 2\theta_{13}$	N_{vac}	N_{mat}^{NH}	N_{mat}^{IH}	σ_{NH-vac}	σ_{NH-IH}
0.05	260	227	264	2.2σ	2.5σ
0.1	261	204	262	4.0σ	4.1σ
0.2	263	163	261	7.8σ	7.7σ

$\sin^2 2\theta_{13}$	N_{vac}	N_{mat}^{NH}	N_{down}	σ_{NH-vac}
0.05	260	227	410	1.8σ
0.1	261	204	410	3.3σ
0.2	263	163	410	6.6σ

Fe Calorimeter, 1000kt-yr, $\Delta\chi^2$ and σ sensitivity . . .

Range 2 : E = 4 - 8 GeV and L = 8000 - 10700 Km
($Log_{10}(L/E)$=3.21-3.44)

$\sin^2 2\theta_{13}$	N_{vac}	N_{mat}^{NH}	σ^{NH-vac}
0.05	23	43	3.0σ
0.1	23	63	5.0σ
0.2	24	104	7.8σ

$\sin^2 2\theta_{13}$	N_{vac}	N_{mat}^{NH}	N_{down}	σ^{NH-vac}
0.05	23	43	156	2.7σ
0.1	23	63	156	4.3σ
0.2	24	104	156	6.1σ

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- Future superbeam experiments using the $\nu_\mu \longrightarrow \nu_e$ channel for hierarchy determination will find it difficult to acheive the required sensitivities due to degeneracies
- It is worthwhile to explore the $\nu_\mu \longrightarrow \nu_\mu$ channel for this purpose since it is largely free of degeneracies.

To Sum Up . . .

- Large matter effects are not only confined to $P_{\mu e}$ but also arise in $P_{\mu\tau}$ at GeV energies at very long Earth baselines. Both must be properly considered when evaluating the event-rates for experiments measuring muon survival.
- The effects discussed above are significantly sensitive to θ_{13} and Sign of Δm^2_{31}, determination of which are outstanding problems of neutrino physics.
- We have tried to show that there is a good possibility that one can determine the Sign of Δm^2_{31} using atmospheric neutrinos in a large mass iron calorimeter with charge identification capabilities.

Matter effect in $\nu_\mu \rightarrow \nu_\tau$

♠ Click here for $P_{\mu\tau} \rightsquigarrow$

◇ The animation shows the matter effects building up in $P_{\mu\tau}$ for L = 6000 to 10500 km.
◇ Maximum matter effects seen at 9700 km.
◇ The term wise break up of probability is also depicted in the animation.

→

To Summary page

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