Matter effects in long baseline experiments

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ork done with Anindya Datta hep-ph/0410266; JHEP)

Oscillations : 3 generation

trino flavour states : $|\nu_\alpha\rangle$ Mass eigenstates $|\nu_i\rangle$

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle \ ,$$

s a 3×3 unitary matrix (Pontecorvo-Maki-Nakagawa-Sakata mixing matrix).

ume there are no Majorana phases.
U parametrized in terms of three mixing angles and a CP-violating phase $\delta_{\rm CP}$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta_{\rm CP}} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta_{\rm CP}} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{pmatrix} ,$$

acuum, oscillations governed by U and two independent differences $\delta m^2_{ij} \equiv m_i^2 - m_j^2$.

ll make a further simplifying assumption : $\delta_{\rm CP} = 0$.

Oscillations : 3 generation

ar neutrino flux measurements, augmented by the KamLAND reactor data, give

$$0.31 < \tan^2\theta_{12} < 0.69, \qquad 7.0 \times 10^{-5}\ \mathrm{eV}^2 < \delta_{21} < 9.3 \times 10^{-5}\ \mathrm{eV}^2$$

t-fit values: $\tan^2\theta_{12} = 0.45$ and $\delta m^2_{21} = 8.0 \times 10^{-5}\ \mathrm{eV}^2$.

nospheric ν (SuperK, SNO) (as well as K2K):

$$\mathbf{\sin^2 2\theta_{23} > 0.86}, \qquad 1.4 \times 10^{-3}\ \mathrm{eV}^2 < |\delta m^2_{32}| < 5.1 \times 10^{-3}\ \mathrm{eV}^2$$

t-fit value: $|\delta m^2_{32}| = 2.0 \times 10^{-3}\ \mathrm{eV}^2$.

gative result of the CHOOZ experiment:

$$\sin^2\theta_{13} < 5 \times 10^{-2} \qquad \text{at 99.73\% C.L.}$$

Oscillations : 3 generation

Jnlike in the case for δm^2_{21}, the sign of δm^2_{32} is undetermined.

$\delta m^2_{32}| \approx |\delta m^2_{31}| \gg |\delta m^2_{21}|$

$\Longrightarrow$ δm^2_{21} & θ_{12} play only a subservient role in ν_μ oscillations;

vhile $|U_{\mu 3}|$ is constrained to be close to $1/\sqrt{2}$, it is still allowed to be non-maximal.

he presence of matter, the effective Hamiltonian is

$$H = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta_{21} & 0 \\ 0 & 0 & \Delta_{31} \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ,$$

ere $A \equiv \pm\sqrt{2} G_F N_e$ for $\overset{(-)}{\nu_e}$.

Vanishingly small δm^2_{21}

illation/survival probabilities can be expressed in terms of the dominant mass-square erence ($\delta m^2_{31} \approx \delta m^2_{32}$) and just two of the mixing angles

a ν_μ ($\bar{\nu}_\mu$) of energy E_ν traveling a distance L in vacuum, the survival probability is

$$P^{\rm vac}_{\mu\mu} = P^{\rm vac}_{\bar{\mu}\bar{\mu}} = 1 - 4\,|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2) \sin^2 (\Delta_{31} L/2) \ ,$$

ere,

$$U_{e3} = s_{13} \ , \qquad U_{\mu 3} = c_{13}\, s_{23} \ , \qquad \Delta_{31} \equiv \frac{\delta m^2_{31}}{2 E_\nu} \ .$$

tter effect ?

ructive to look at simplified form :

matter density small enough that A can be treated as a perturbation in $H_{\rm eff}$

(e.g. Sandhya Choubey and Probir Roy)

Small matter densities

his limit,

$$P^{M}_{\mu\mu\,(\bar\mu\bar\mu)} \simeq P^{\rm vac} \pm (\Delta P_{\mu\mu}/2) + \mathcal{O}(\widetilde{A}^2)$$

$$\Delta P_{\mu\mu} \simeq 4\,\widetilde{A}\,|U_{e3}|^2\,|U_{\mu 3}|^2\,(\mathbf{1} - \mathbf{2}\,|\mathbf{U}_{\mu 3}|^2)\,\left[4\sin^2\Psi - \Psi\,\sin(2\,\Psi)\right] + \mathcal{O}(\widetilde{A}^2)$$

ere

$$\widetilde{A} \equiv \frac{A}{\Delta_{31}} \qquad \text{and} \qquad \Psi \equiv \frac{\Delta_{31}\,L}{2}\ .$$

e probability difference (**asymmetry**) $\Delta P_{\mu\mu}$ is a measure of the matter effect.

- proportional to both $\widetilde{A}$ and $|U_{\mu 3}|^2$.

and ν_τ have the same interaction with matter

- any matter effect can only seep in through a mixing with the ν_e.
- overall factor of $|U_{e3}|^2$.

these three proportionalities independent of the approximation, and are exact results.

Small matter densities

ctor $(1 - 2|U_{\mu 3}|^2)$: consequence of the approximation of a small matter term $\widetilde{A}$

applicable only for neutrinos traversing small base lines.

Vanishes identically for a maximal mixing in the $\nu_\mu - \nu_\tau$ sector

$\Delta P_{\mu\mu}$ is potentially a sensitive probe of the maximality of $U_{\mu 3}$.

nvolute survival probabilities with the E_ν-dependent flux and efficiency of a given ector

the number of events and thereby the event asymmetry

$$\Delta N = N_{\nu_\mu} - N_{\bar{\nu}_\mu} \equiv N_- - N_+ \ .$$

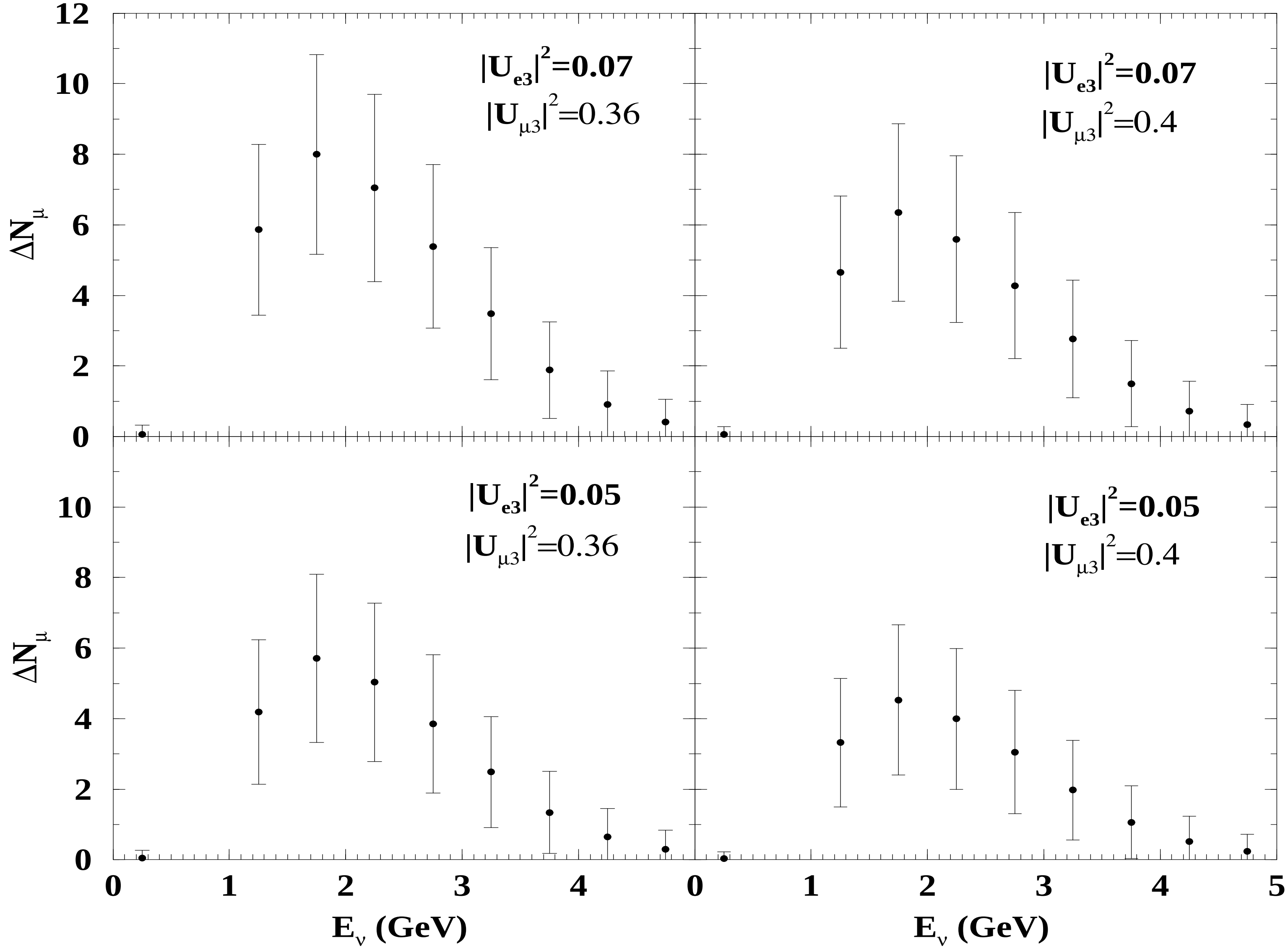
here ??

utrino source : Fermilab Main Injector   Detector : MINOS at the Soudan Mine

eline : 732 km and neutrinos skim only the earth's surface (low density)

NuMI–MINOS

m : with a ν_μ exposure of 16×10^{20} primary protons on target (Choubey & Roy)



owever, error handling
as not right.

ven using an optimistic
dicator of the
ckground fluctuation,
r the given luminosity,
ch of the rate
ymmetries consistent
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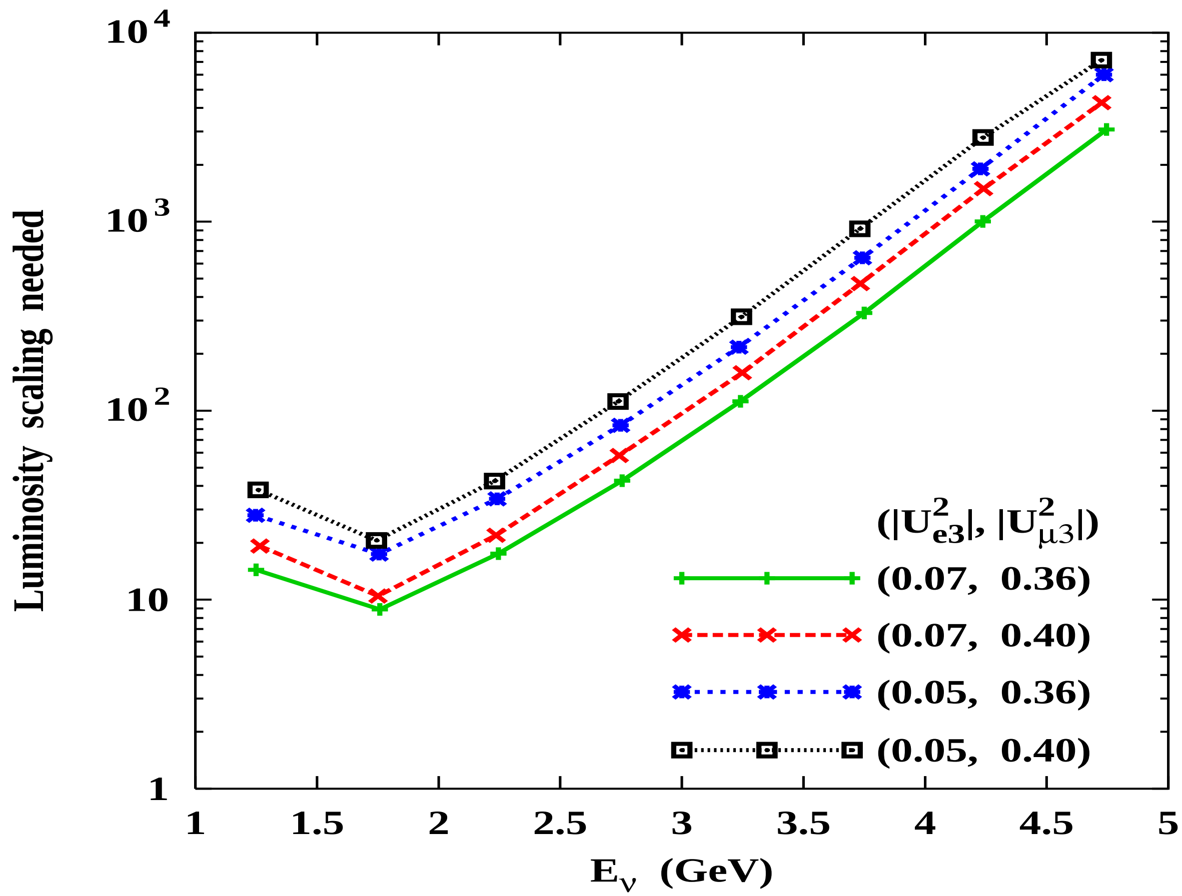
assive scaling of
minosity needed



s projection altogether neglects the systematic uncertainties.

eral uncertainties involved in the translation between the visible energy observed at a ector and the energy of the neutrino to which the event is ascribed.

Jncertainties in the assumed final state multiplicities

he scattering or absorption of the secondary particles etc.

NERνA collaboration:
of the corresponding systematic effects at MINOS-like experiments could be nparable to, or even dominate, the statistical errors.

at about longer baselines ?

Matter effects get time to build up.

Larger densities faced (earth core !)

all $\widetilde{A}$ approximation does not work !

Double Series Expansion

arbitrarily large but constant density, the survival probability is

$$\mu\mu \quad = \quad 1 - 4\, s_{23}^2 \left(c_{23}^2 s_{\theta_m}^2 \sin^2 \left[\frac{\Delta_{31} L}{4} \left(1 + \widetilde{A} - \mathcal{D} \right) \right] \right.$$
$$\left. + s_{23}^2 s_{\theta_m}^2 c_{\theta_m}^2 \sin^2 \frac{\Delta_{31} L \mathcal{D}}{2} + c_{23}^2 c_{\theta_m}^2 \sin^2 \left[\frac{\Delta_{31} L}{4} \left(1 + \widetilde{A} + \mathcal{D} \right) \right] \right) ,$$

ere

$$\mathcal{D} \equiv \sqrt{1 + \widetilde{A}^2 - 2\, \widetilde{A} \cos 2\theta_{13}} \ , \quad \text{and} \quad \theta_m \equiv \frac{1}{2} \tan^{-1} \frac{\sin 2\theta_{13}}{\cos 2\theta_{13} - \widetilde{A}} \ .$$

form a double expansion in $|U_{e3}|$ as well as $\beta \equiv \frac{1}{2} - |U_{\mu 3}|^2$

w for any value for $\widetilde{A}$.

Double Series Expansion

ining $\quad \Omega \equiv \frac{\Delta_{31}\, L\, \widetilde{A}}{2} = \frac{A\, L}{2} = \widetilde{A}\, \Psi \qquad \text{and} \qquad \Theta \equiv \Omega - \Psi \ .$

$$
\begin{aligned}
{}_{\mu} \quad &= \left(1 - 4\,\beta^2\right) c_\Psi^2 \\
&+ \frac{|U_{e3}|^2}{\left(1 - \widetilde{A}\right)^2} \left[\left\{ s_\Psi^2 - s_\Omega^2 - s_\Theta^2 - \widetilde{A}\left(1 - \widetilde{A}\right) \Psi\, s_{2\Psi} \right\} - 4\beta \left\{ (1 - \widetilde{A})^2\, s_\Psi^2 - s_\Omega^2 \right\} \right] \\
&+ \mathcal{O}(|U_{e3}|^4, |U_{e3}|^3\beta, |U_{e3}|^2\beta^2, |U_{e3}|\beta^3) \ ,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\left(1 - \widetilde{A}^2\right)^3 \Delta P}{\widetilde{A}\, |U_{e3}|^2\, |U_{\mu 3}|^2} \quad &= \left(1 - \widetilde{A}^2\right) \left[4\, c_\Psi^2\, s_\Omega^2 + (1 - \widetilde{A}^2)\, \Psi\, s_{2\Psi} - \frac{1 + \widetilde{A}^2}{2\, \widetilde{A}}\, s_{2\Psi}\, s_{2\Omega} \right] \\
&+ 2\,\beta\,(1 - \widetilde{A}^2) \left[-\,4\, s_\Psi^2\, s_\Omega^2 + (1 - \widetilde{A}^2)\, \Psi\, s_{2\Psi} + \frac{1 + \widetilde{A}^2}{2\, \widetilde{A}}\, s_{2\Psi}\, s_{2\Omega} \right] \\
&+ \mathcal{O}(|U_{e3}|^2, |U_{e3}|\beta, \beta^2) \ .
\end{aligned}
$$

Double Series Expansion

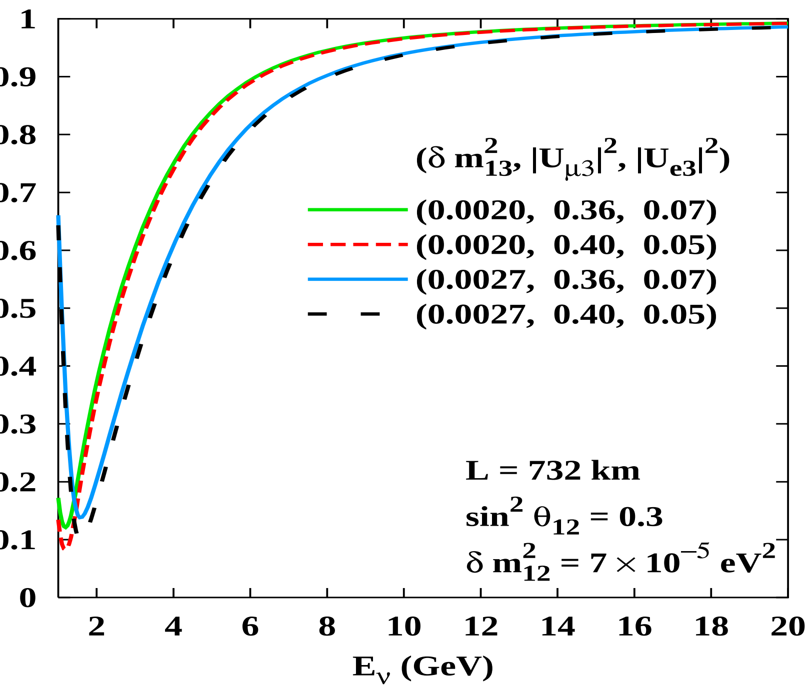
To $\mathcal{O}(\widetilde{A})$, the expressions for both $P_{\mu\mu}$ and ΔP coincide with earlier ones.

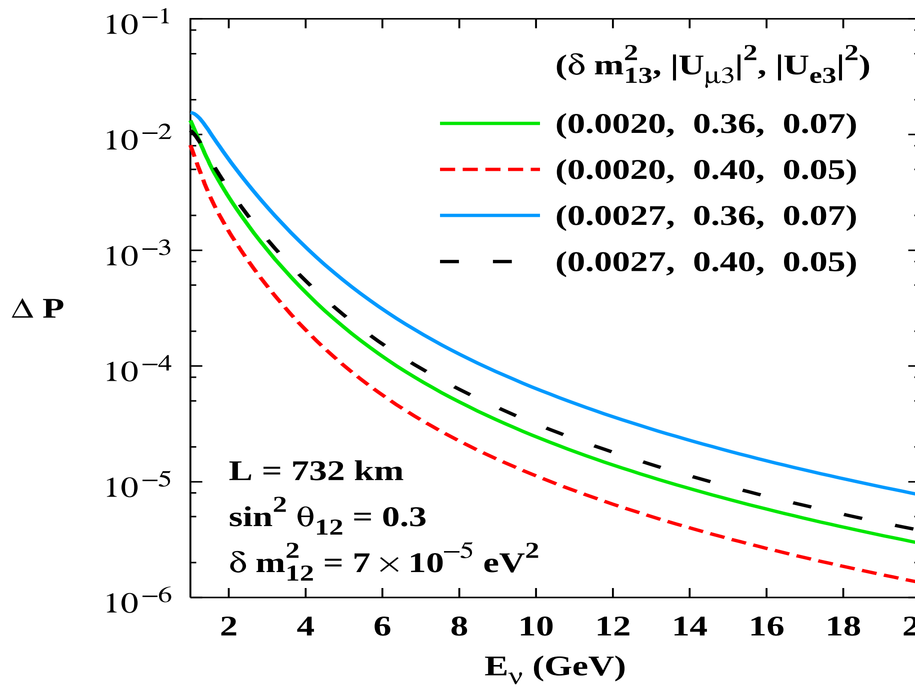
The probability asymmetry ΔP continues to be proportional to each of $\widetilde{A}$, $|U_{e3}|^2$ and $|U_{\mu 3}|^2$, but not to $\beta \equiv (1 - 2|U_{\mu 3}|^2)$.

The strength of the $\mathcal{O}(1)$ term on the r.h.s. —which vanishes as $\widetilde{A}^2$ for small $\widetilde{A}$— gives a measure of the violation of the $\mathbf{(1 - 2|U_{\mu 3}|^2)}$ proportionality of ΔP.

MINOS again!

ortant quantities : ΔP and $P_{\rm av} \equiv (P_{\mu\mu} + P_{\bar\mu\bar\mu})/2$. [significance $\propto \Delta P_{\mu\mu}/\sqrt{2P_{\rm av}}$].





' very small, especially at larger neutrino energies.

claim signal based on $\Delta P \lesssim 10^{-4}$

accurate knowledge of both cross sections as well as density profile within the earth.

MINOS again!

- ncentrate on $E_\nu \lesssim 2$ GeV??
- he asymmetry is not too large even in this range;
- necessitates both a very good energy resolution as well as a low energy threshold.
- $\sigma \propto E_\nu^2$; thus restricting ourselves to a small window is tantamount to rejecting a very arge fraction of the events.

The case for a very long baseline

e size of ΔP improves dramatically with an increase in the baseline.

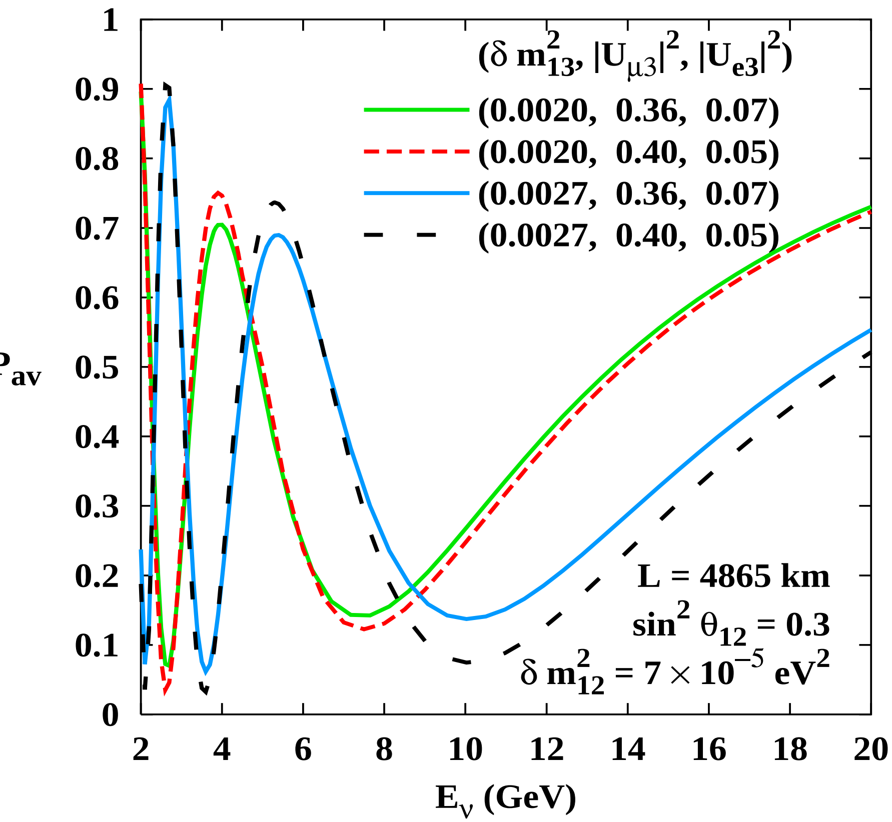
definiteness, shall consider

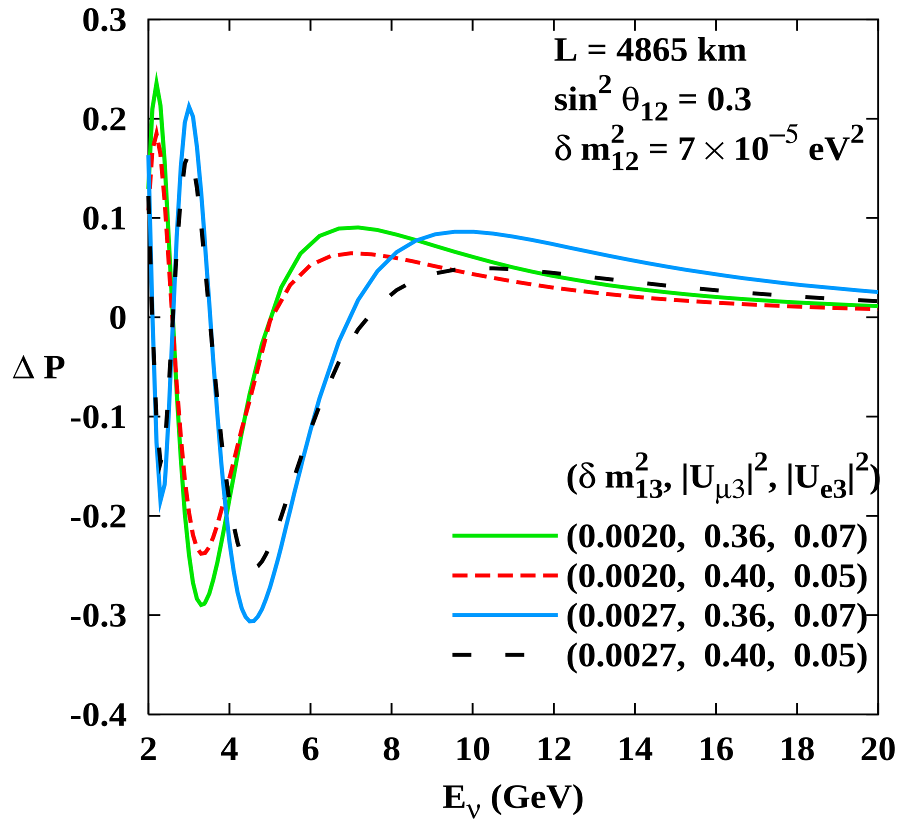
$L = 4865$ km :	distance between the Japan Hadron Facility and the proposed Indian Neutrino Observatory site at Rammam
$= 10480$ km :	distance between Fermilab and INO.

tter density varies quite a bit.

numerical work : use the full expression for neutrino propagation in matter with the sity profile being given by the Preliminary Reference Earth Model.

The case for a very long baseline



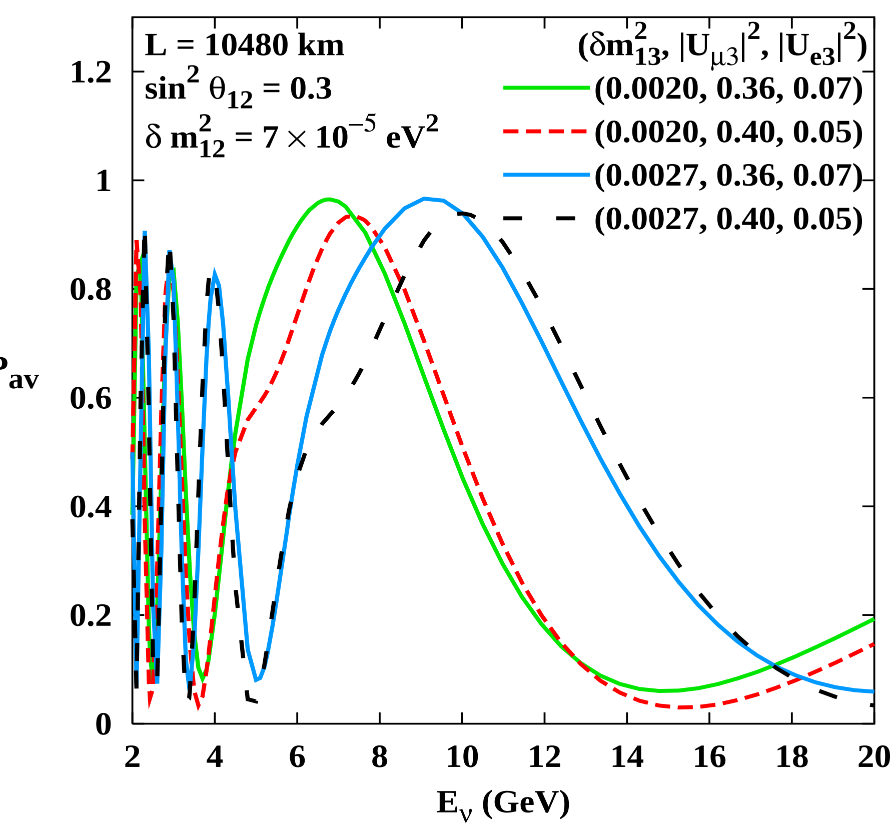


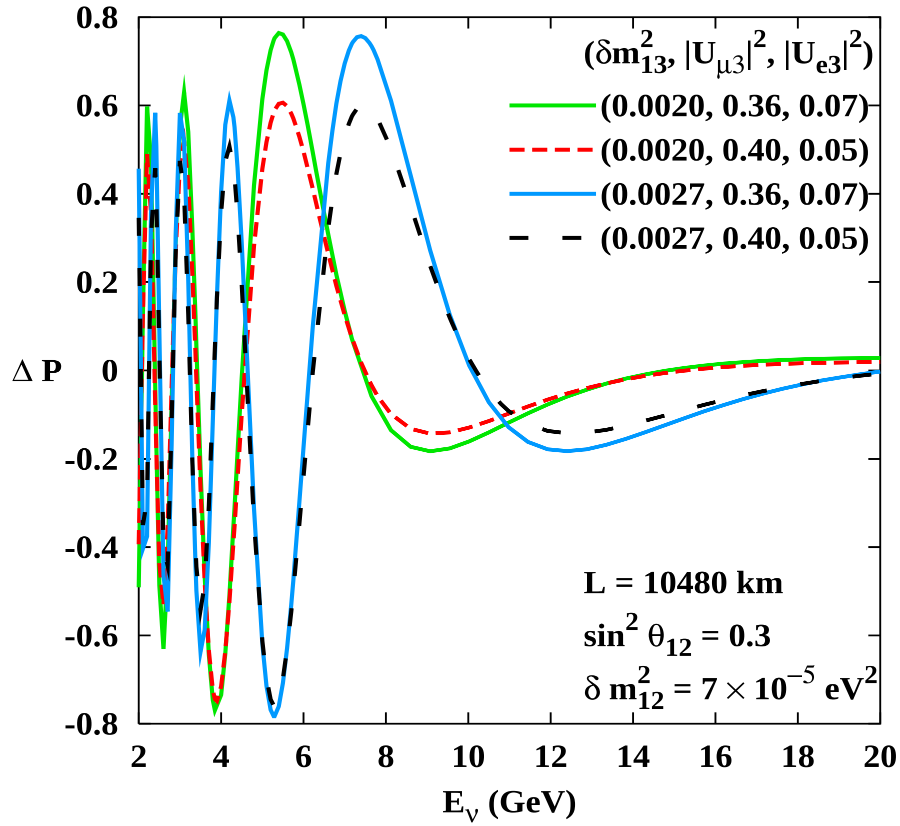
ΔP is large over a wide range of E_ν.

ignificant structure, including sign reversal, in ΔP

or 732 km baseline, ΔP was nearly monotonic in E_ν,

The case for a very long baseline





$L = 10480$ km case looks even better !

Position of the maxima of the survival probabilities as well as the difference ΔP has some dependence on δm^2_{13}.

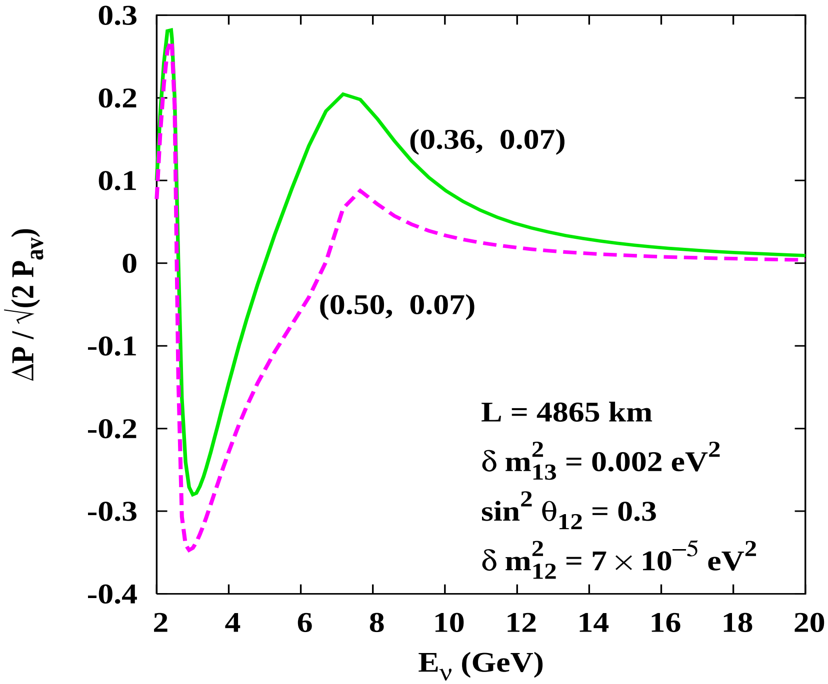
eat !!

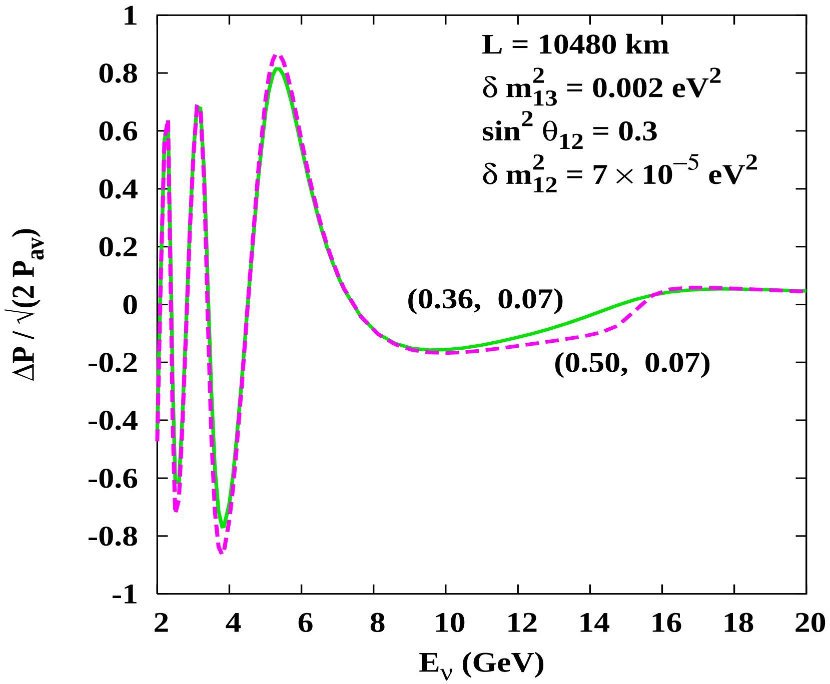
The case for a very long baseline

Deviation of $|U_{\mu 3}|$ from maximality not the only source of asymmetry.

he higher order terms in $\widetilde{A}$ are not proportional to $(1 - 2|U_{\mu 3}|^2)$.

numerical importance of such contributions naturally increases with the baseline,





For the larger baseline, the bulk of the effect is due to such "higher-order" terms.

The case for a very long baseline

$\Longrightarrow$ while a baseline of ≈ 5000 km may allow for a determination of $(1 - 2|U_{\mu 3}|^2)$ hrough measurements of the rate asymmetry,
for much longer baselines, the sensitivity reduces quite sharply

A measurement of the maximality of $|U_{\mu 3}|$ is not very straightforward for ultra-long baselines.

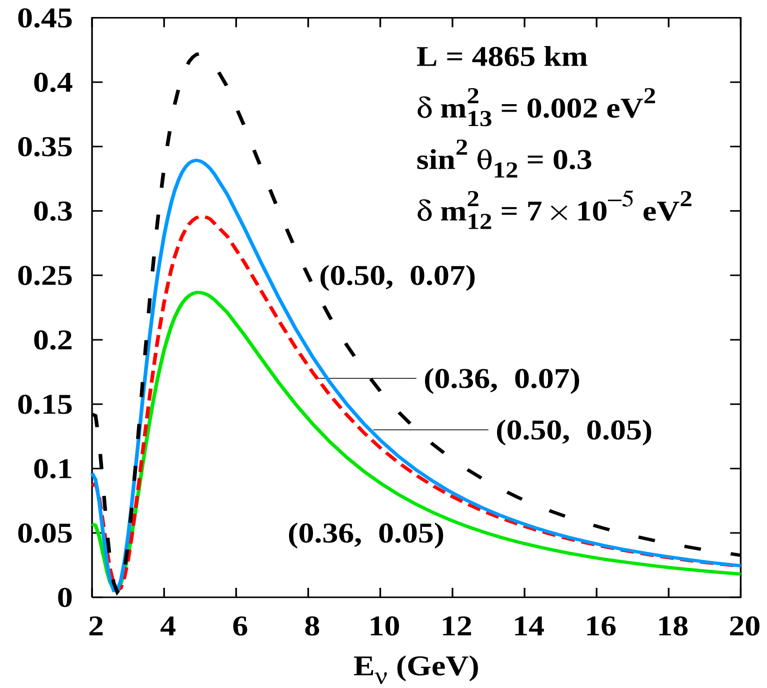
he asymmetry must remain proportional to $|U_{e3}|^2$ even on the inclusion of $\widetilde{A}^n$ terms.

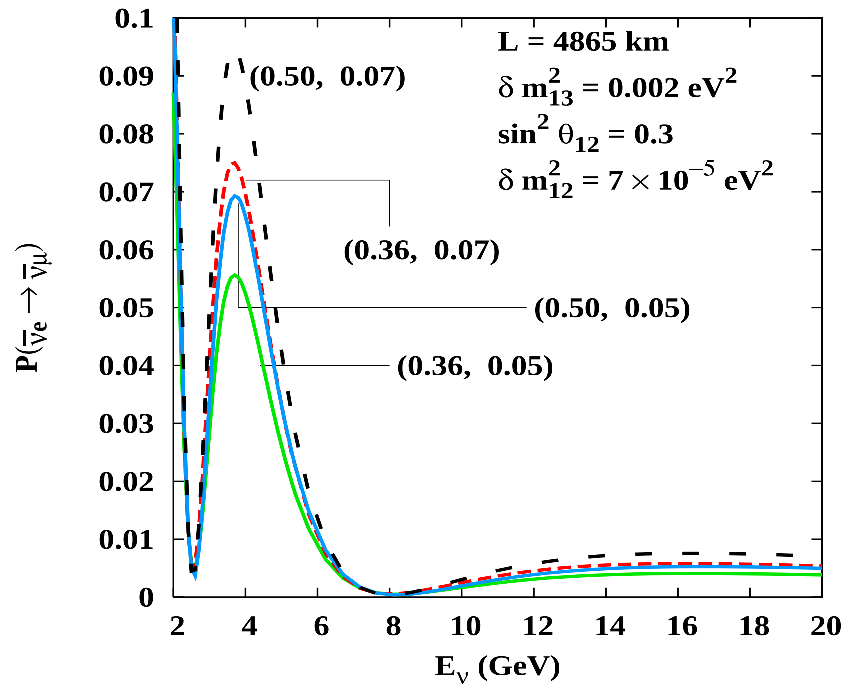
$\Longrightarrow$ $L \sim 10^4$ km may serve the purpose of affording a good measurement of $|U_{e3}|$

Since $U_{\mu 3}$ and U_{e3} inextricably linked even for the case of the 4865 km baseline, such an independent measurement has its own importance.

tter effect seen to play important role in $P_{\mu\mu}$ What about $P(\nu_e \to \nu_\mu)$?

atever source produces $\overset{(-)}{\nu_\mu}$ (beam dump or muon storage ring), would produce $\overset{(-)}{\nu_e}$



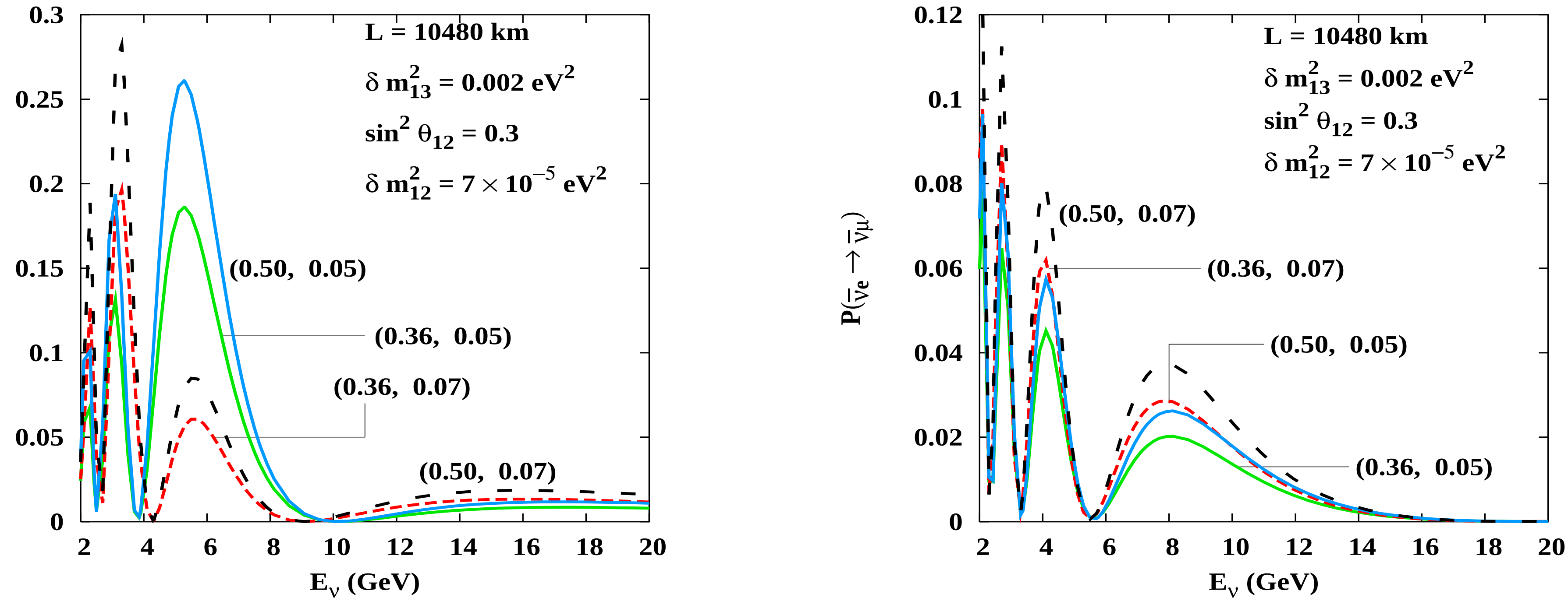


The transition probability $P(\nu_e \to \nu_\mu)$ can be quite sizable.

For $E \gtrsim 3$ GeV, $P(\bar\nu_e \to \bar\nu_\mu) \lesssim P(\nu_e \to \nu_\mu)$.

Detection efficiency for $\bar\nu$ generically smaller. More profitable to work with ν_e than $\bar\nu_e$.

$\nu_e \to \nu_\mu$ oscillations



Transition probabilities have non-trivial dependence on both U_{e3} and $U_{\mu 3}$.

Unlike in the case of ΔP, these, in general, are not proportional to $|U_{e3}|^2$.

The measurement of this effect would thus lead to a constraint in the U_{e3}–$U_{\mu 3}$ plane independent of that drawn from the asymmetry measurement.

Event rate calculation

ıtrino flux from a muon storage ring can be calculated very precisely.

rting with a μ^- beam, the number of μ^- events in a far detector :

$$N_\mu = N_n \int \sigma(\nu_\mu + N \to \mu^- + X) \, \frac{dN_\nu}{dE_{\nu_\mu}} \, P_{\mu\mu}(L, E) \, dE_{\nu_\mu}$$

, : total number of nucleons present in the fixed target. Similarly for μ^+ events.

e $\bar{\nu}_e$ from muon decay could also oscillate into $\bar{\nu}_\mu$ while traversing through the earth result in muonic events.

wrong sign muons and can be easily distinguished in a magnetized detector.

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ll consider a storage ring with 20 GeV muons and a 50 kT Iron calorimeter detector n as the one proposed for MONOLITH or the ICAL/INO experiments.

---

jected energy threshold for muon detection is about 2 GeV

olution is expected to be better than 0.5 GeV over the entire range.

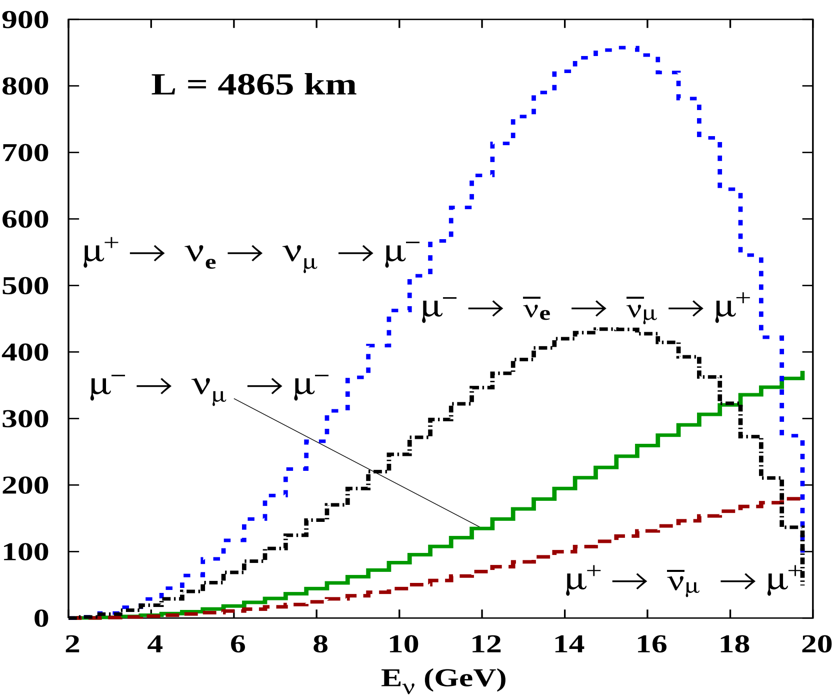
Event rate calculation

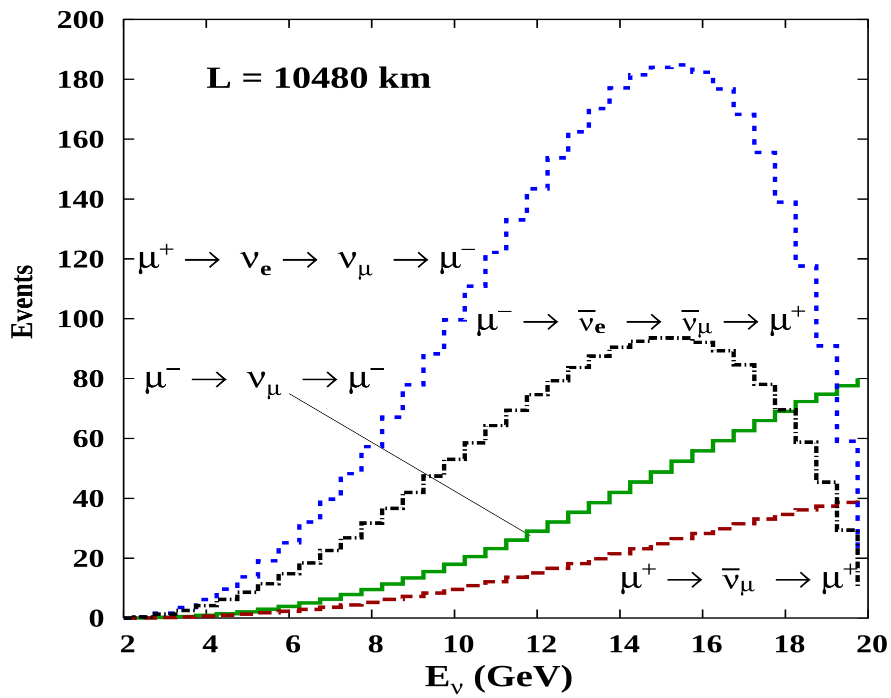
$\Delta P_{\mu\mu}$ typically grows with the baseline. Does sensitivity too ?

Sensitivity of the asymmetry to $(1 - 2\,|U_{\mu 3}|^2)$ reduces for very large baselines, while the sensitivity to $|U_{e3}|^2$ is retained.)

For fixed detector size, solid angle subtended decreases with increasing baseline. $\Longrightarrow$ the neutrino flux goes down quadratically with the baseline

f the survival probability were to be unity,





Event rate calculation

For our numerical results, we assume 10^{20} muon decays per year.

Could instead have started with a higher energy for the muon beam.
For example, the neutrino beam from a 50 GeV storage ring is collimated sufficiently enough to get a similar significance with only 10^{19} muon decays per year.

For long baselines, the survival/oscillation probabilities are very sensitive functions of E_ν, especially for low energies.

Extracting any information from the low energy tail of the muon spectrum would necessitate very good energy resolution.

Detector threshold is high ! Also, cross sections small for such E_ν. So only a relatively small contribution to the total number of muon events.

$\nu - N$ cross-section is nearly double that of $\bar{\nu} - N$.
Detector should be exposed to a $\bar{\nu}_\mu$ beam for double the time that it is exposed to a ν_μ beam.

50 kT-year exposure to a ν_μ beam and a 100 kT-year exposure to $\bar{\nu}_\mu$ beam, or in other words, a total exposure of 3 years for the detector configuration.

Muon detection

use a muon detection threshold of 2 GeV.

eal life, threshold is never a step function.

listic estimation possible only in the context of a particular detector.

comment based only on the results of the detector simulations and prototype studies.

eshold for the charged-current interaction is determined by the number of hits the ergent muon makes in the detector.

pendent on

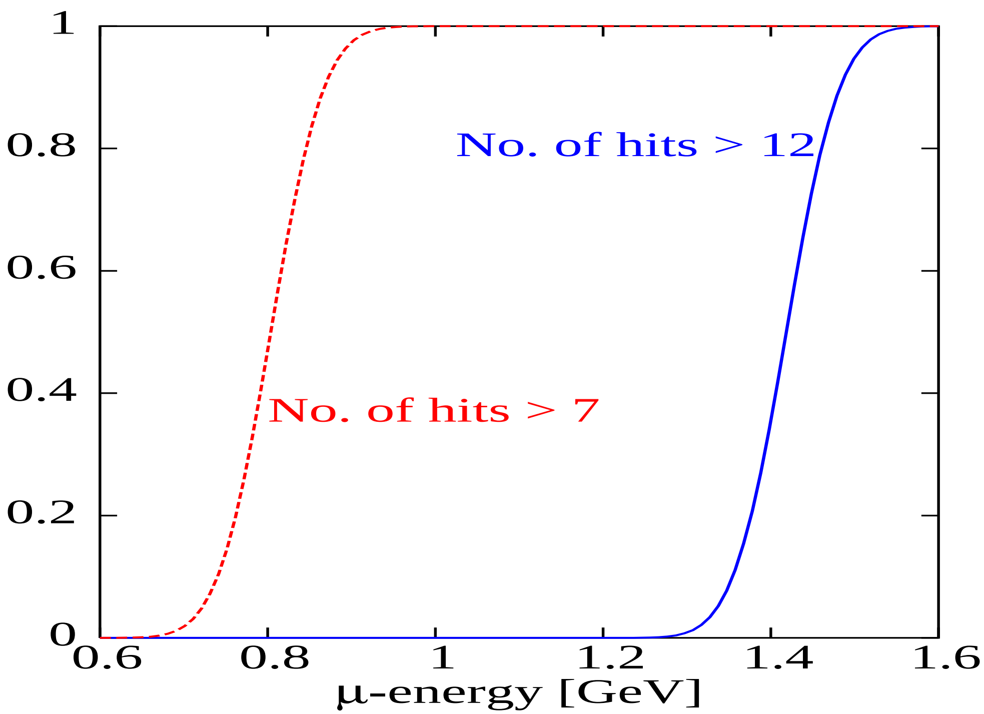
- he energy and angle of the muon;
- he geometry of the detector (alongwith the position of the initial interaction point within the detector);
- he number density of the active detector components

Muon detection

ailed detector simulations of ICAL/INO collaboration:

umber of hits well approximated by a Gaussian distribution with
mean $\propto E_\mu$ and the width virtually independent of it.

seven hits in the detector constitutes an observation



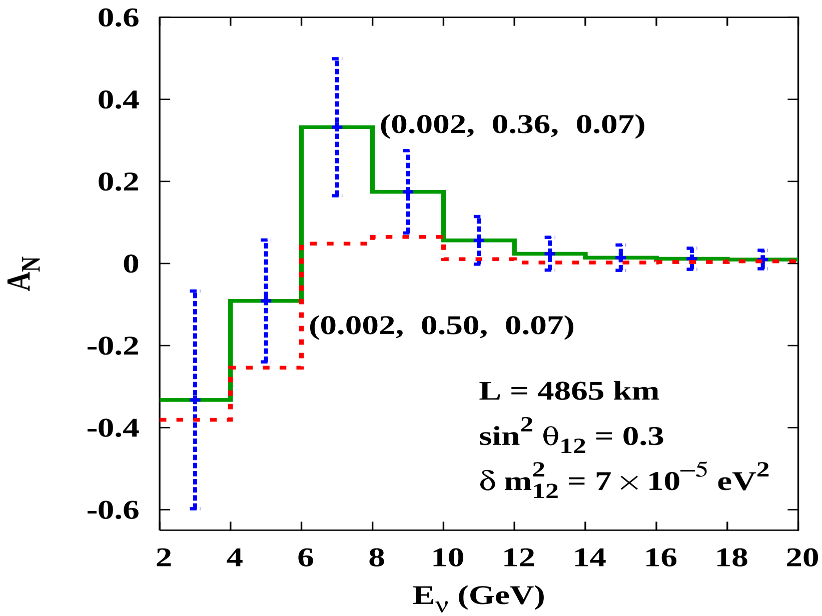
For 7 hits, $E_\mu > 1$ GeV $\Longrightarrow$ probability of detection is ≈ 1

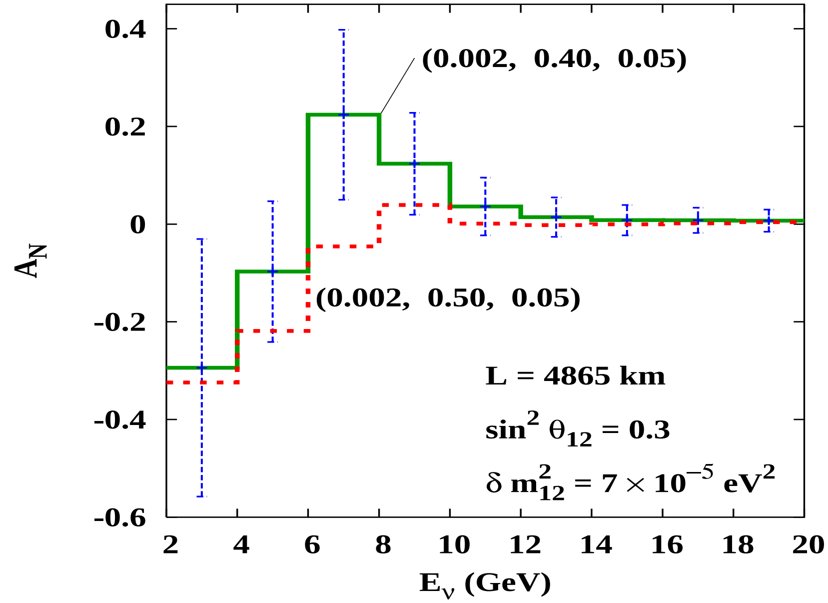
If we demand a minimum of 12 hits in the detector to constitute an observation,
threshold shifted to ≈ 1.5 GeV

Rate asymmetry for $\nu_\mu(\bar{\nu}_\mu)$ initiated events

ine: $A_N \equiv (N_- - N_+)/(N_- + N_+)$

the expected data, keeping under consideration both the expected energy resolution vell as the number of events in a particular bin



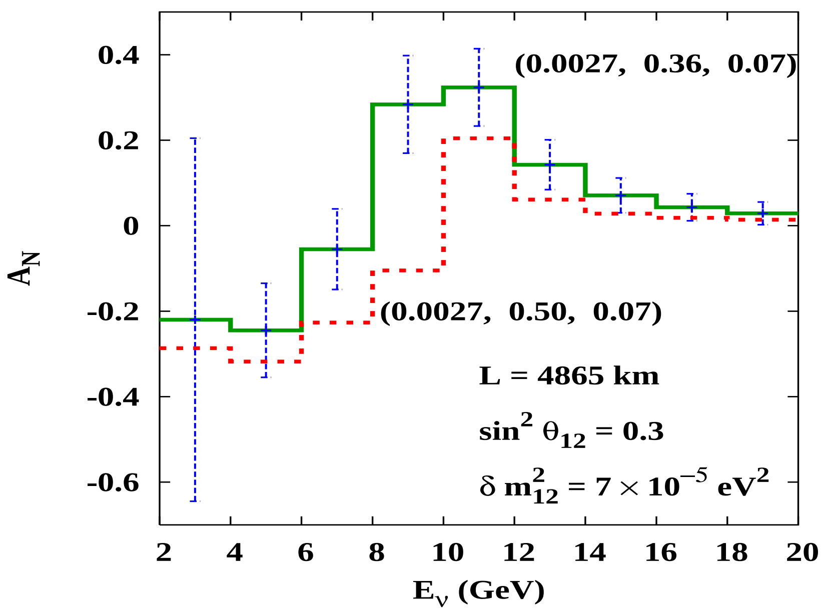


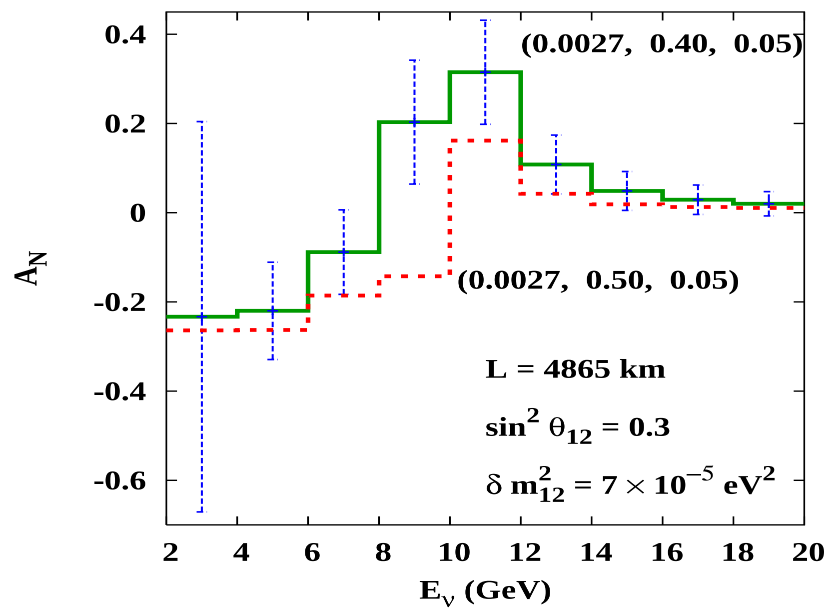
The rate asymmetry is statistically significant in more than one bin. Demonstrates the ensitivity of such an experiment to a departure from maximality of $U_{\mu 3}$.

Rate asymmetry for $\nu_\mu(\bar{\nu}_\mu)$ initiated events

Signal less pronounced for $(|U_{\mu 3}|^2, |U_{e3}|^2) = (0.40, 0.05)$ than for $(0.36, 0.07)$.

Reflection of $\Delta P \propto |U_{e3}|^2$. Proportionality to $(1 - 2\,|U_{\mu 3}|^2)$?





n the range of interest, the bin wise asymmetry shows significant energy-dependence hint of oscillation?)

Would have been absent for a similar detector had the baseline been shorter than ~ 1000 km.

Rate asymmetry for $\nu_\mu(\bar{\nu}_\mu)$ initiated events

$\Delta N(E_\nu)$ exhibits a discernible dependence on $(|U_{\mu 3}|^2, |U_{e3}|^2)$.
But, significant resolution in the parameter space, needs larger statistics.

Dependence on δm^2_{13} much more pronounced.
Shift in ΔP well reflected by a shift in A_N, even after the convolution with

- the muon spectrum
- the energy-dependent cross sections
- the finite resolution effects.

Thus, this measurement could be used for a determination of δm^2_{13}.

Energy dependence is quite different for the two baselines considered.
If storage rings come up at both the JHF and Fermilab, such a detector could use beams from both to distinguish more efficiently between the possible parameter sets.

Using ratio A_N rather than $(N_- - N_+)$ eliminates much of the systematic errors.
But A_N is not immediately reflective of the actual number of events.
Apparently large A_N values for small E_ν bins have larger error bars and hence of relatively low statistical significance.

Wrong sign muons

ıtrino beam in a storage ring : $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$

$\longrightarrow \nu_\mu \quad \& \quad \nu_\mu + N \to \mu^- + N' \qquad vs. \qquad \bar{\nu}_e \longrightarrow \bar{\nu}_\mu \quad \& \quad \nu_\mu + N \to \mu^+ + N'$

earance of a muon with a charge opposite to that in the storage ring.

ırge measurement is relatively straightforward in a magnetized detector.

ong charge muons have been considered for θ_{13} measurement.

ı these also help in a more accurate measurement of θ_{23}?

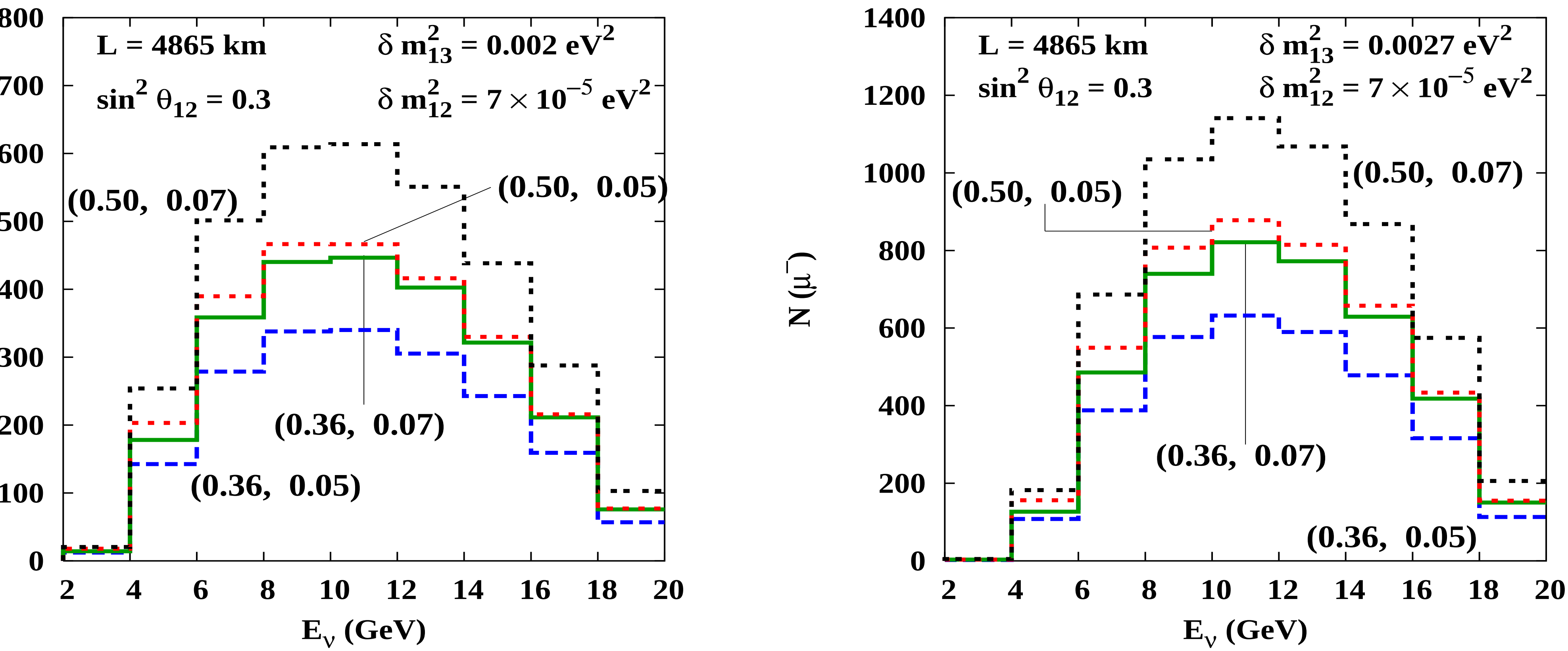
ically $P(\nu_e \to \nu_\mu) > P(\bar{\nu}_e \to \bar{\nu}_\mu)$, a consequence of the $sgn(\delta m^2_{12})$.

reover, smaller detection efficiency for $\bar{\nu}_\mu$

concentrate on the $\mu^+ \to \nu_e \to \nu_\mu \to \mu^-$ chain (forget about conjugate).

ve advocated twice as long an exposure to μ^+ decays as compared to μ^- decays!

μ^- events from μ^+ decays



Number of events grows with $|U_{e3}|^2$. (Not exactly linear, but close to).
Also the case for the larger baseline of 10480 km.

Dependence on $|U_{\mu 3}|$ more complicated though. Depends more crucially on the size of the matter effect and hence on the exact baseline.

μ^- events from μ^+ decays

Given the large number of such events, this measurement is likely to be a sensitive probe in the $(|U_{\mu 3}|, |U_{e3}|)$ plane.

Unfortunately, the event distribution is not very sensitive to δm^2_{13}.

measurement of charge ??
ile charge identification in a magentized detector is theoretically straightforward, in ity there is always a possibility for misidentification.
ne wrong sign muon events would be identified as right sign ones and vice versa.

Mismeasurement of charge ??

note Prob(misid.) $= \mathcal{P}$ $\Longrightarrow$ purity of charge measurement $= 1 - \mathcal{P}$)

rting from a beam of decaying μ^-, the number of events in the detector that would *identified* as containing a μ^- is now

$$N^{\rm raw}_{-\,;\,-} = (1 - \mathcal{P})\; N(\mu^- \to \nu_\mu \to \mu^-) + \mathcal{P}\; N(\mu^- \to \bar\nu_e \to \bar\nu_\mu \to \mu^+)$$

analogous expression obtains for $N^{\rm raw}_{+\,;\,+}$.

$$\text{For the asymmetry} \qquad A_N \to A_N^{\rm raw} \equiv \frac{N^{\rm raw}_{-\,;\,-} - N^{\rm raw}_{+\,;\,+}}{N^{\rm raw}_{-\,;\,-} + N^{\rm raw}_{+\,;\,+}} \; .$$

ce $N(\mu^+ \to \nu_e \to \nu_\mu \to \mu^-)$ is typically larger than the rate for the conjugate cess, the main effect on $N^{\rm raw}_{-\,;\,-}$ is just a relative scaling by the factor $(1 - \mathcal{P})$.

$^{\rm w}_{;\,+}$ receives a more nontrivial correction.

L/INO : expect $\mathcal{P} \lesssim 0.07$ for very high efficiencies
n in the first phase, and likely to be significantly better at later stages.

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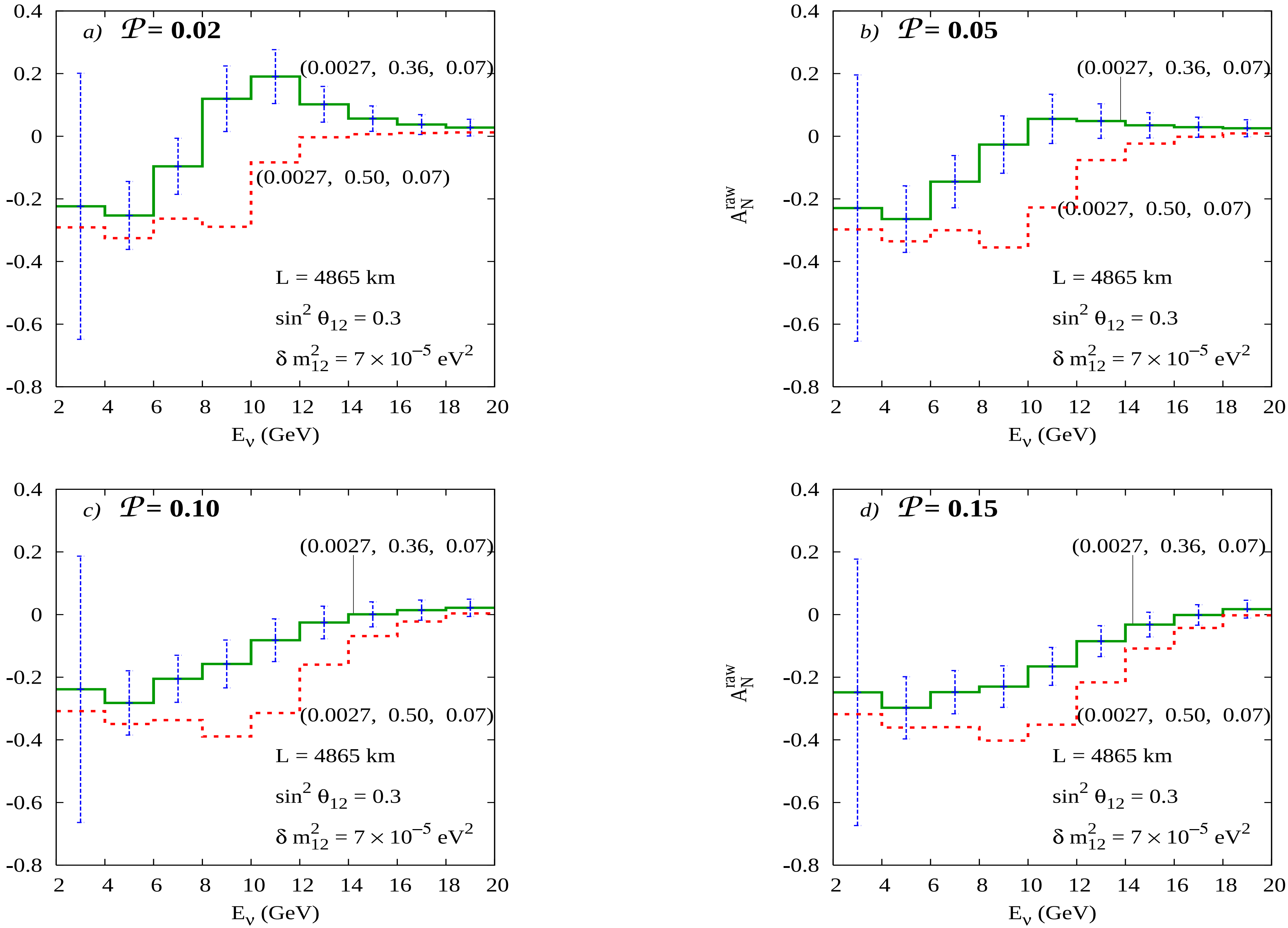

Conclusions

Traditionally, analyses of matter effects in ν_μ oscillations have largely concentrated on transition probabilities.

We demonstrate that the survival probabilities too are very sensitive to matter effects and can be used profitably in determination of crucial parameters in the neutrino sector.

We derive a set of approximate analytical expressions for $P_{\mu\mu}$ in presence an arbitrarily large matter density. (A very wide range of validity.)

Concentraing on CC events, starting with ν_μ and $\bar{\nu}_\mu$ beams,
asymmetry $A_N \equiv (N_- - N_+)/(N_- + N_+)$ is a good measure of the matter effects felt by the neutrinos while propagating through the earth.
However, contrary to claims in the literature the size of the asymmetry for the Fermilab-MINOS combine is much smaller than the experimental sensitivity.

We suggest that a future long-baseline experiment could explore this effect.

Conclusions

- As a prototype experiment, we consider a neutrino factory and the proposed 50 kT ron calorimeter detector with a capability of muon charge determination.

- Using realistic experimental resolutions, we demonstrate that such an experiment can establish the aforementioned asymmetry.

- However, for a long baseline, the asymmetry is no longer proportional to the deviation of $|U_{\mu 3}|^2$ from maximality.

- Even so, the sensitivity to $(1 - 2\,|U_{\mu 3}|^2)$ remains quite pronounced for baselines of upto about 6500 Km.

- For much longer baselines, asymmetry data insensitive to $(1 - 2\,|U_{\mu 3}|^2)$, but can still be used for precise determination of $|U_{e3}|$.

- Measurement of any of the small parameters ($|U_{e3}|$ or β) at long baseline experiments cannot be done independent of each other.

- t is here that the wrong-sign muon rates (which is more sensitive to U_{e3}) have the most important role to play.