
Modeling Neutrino Nucleus Scattering Cross Section in the few GeV region

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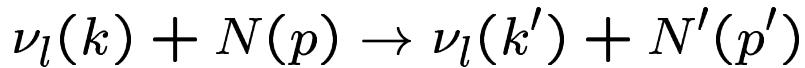
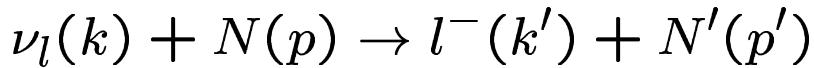
S.Ahmed

One of the major inputs in predicting the neutrino event rates in the neutrino oscillation experiments is

"neutrino nucleus reaction cross sections".

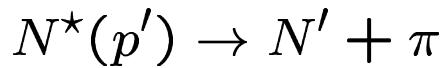
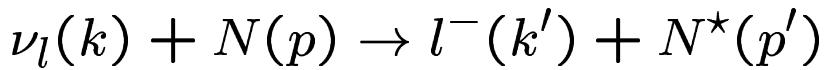
Major contributions to the $\nu - A$ cross sections in the few GeV region($< 3\text{GeV}$) are from the following processes

1. Quasielastic Reaction

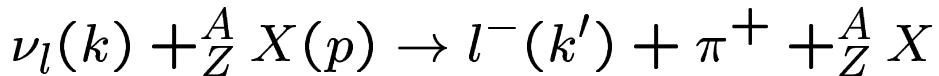


2. Pion Production through Baryon Resonances

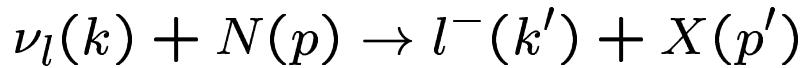
i) Incoherent Pion Production



ii) Coherent Pion Production



3. Deep Inelastic Reaction



All the neutrino event generators use some nuclear model to estimate σ but is limited only to Q. E. reactions

For example in the case of Charged Current Reaction

Common Theoretical Inputs to all ν Event Generators:

- Llewellyn Smith free nucleon QE x-section
- Rein and Sehgal Resonance x-section
- Standard DIS formula for high W, Q^2 .

Inputs which are different for various ν Event Generators:

- Treatment of Nuclear Effects
- Joining of Resonance and DIS
- Treatment of FSI

Neutrino Event Generators

NUANCE

In nuance q.e. scattering comprises both c.c. and n.c. neutrino interactions with nucleons.

- Relativistic Fermi gas model of Smith and Moniz.

NUANCE v2: Dipole form factors, $M_v = 0.84$, $M_A = 1.0 GeV$

NUANCE v3: Non-dipole form factors, π absorption model tuned on π data and $M_A = 1.03 GeV$.

Resonance Processes

Rein and Sehgal model is used.

Nuclear medium effects have not been taken for Resonance Production

An ad hoc suppression of pion production:

20% for $I_3 = \pm \frac{1}{2}$ Resonance Excitations

10% for $I_3 = \pm \frac{3}{2}$ Resonance Excitations

Coherent Processes

Rein and Sehgal model is used.

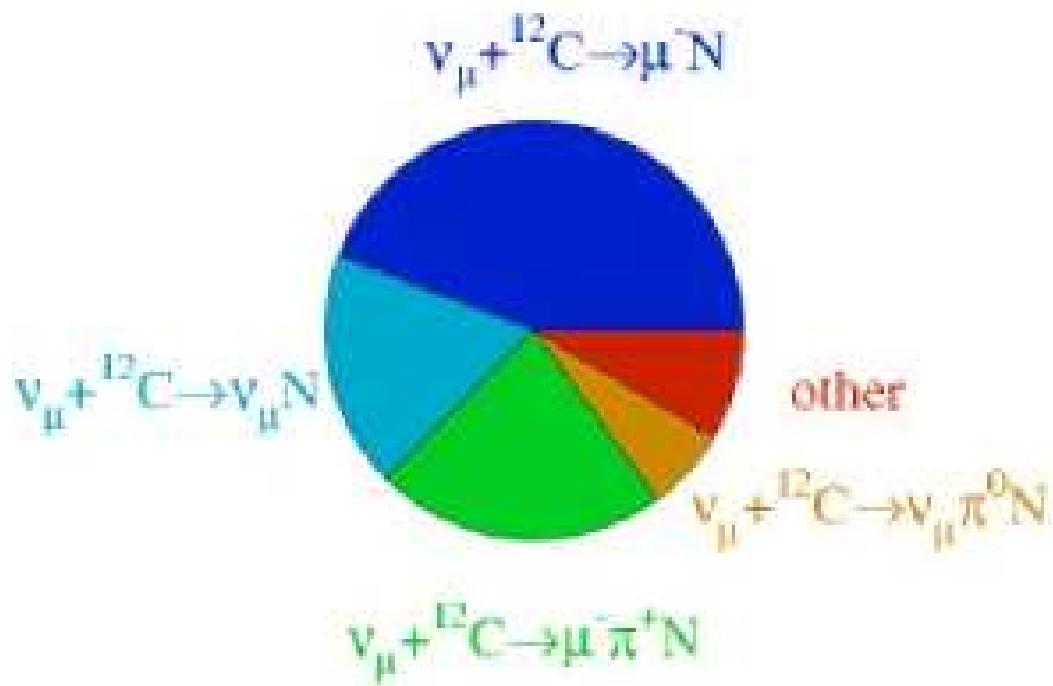
Nuclear Medium Effects on π Production is included through effective π -Nucleus Scattering

Deep Inelastic Scattering

- Parton Distribution Function of Albright and Jarlskog
- Nuclear Medium Effects
- $W > 2GeV$

The relative importance of various processes depend upon the energy range of the neutrinos.

For example, in the case of MiniBooNE experiment, the relative contributions for the various processes are:



C. C. Q. E. Events 40%, C. C. Res. Prod. Events 25%

N. C. E. Events 7%, N. C. π^0 Events 7%

NEUGEN

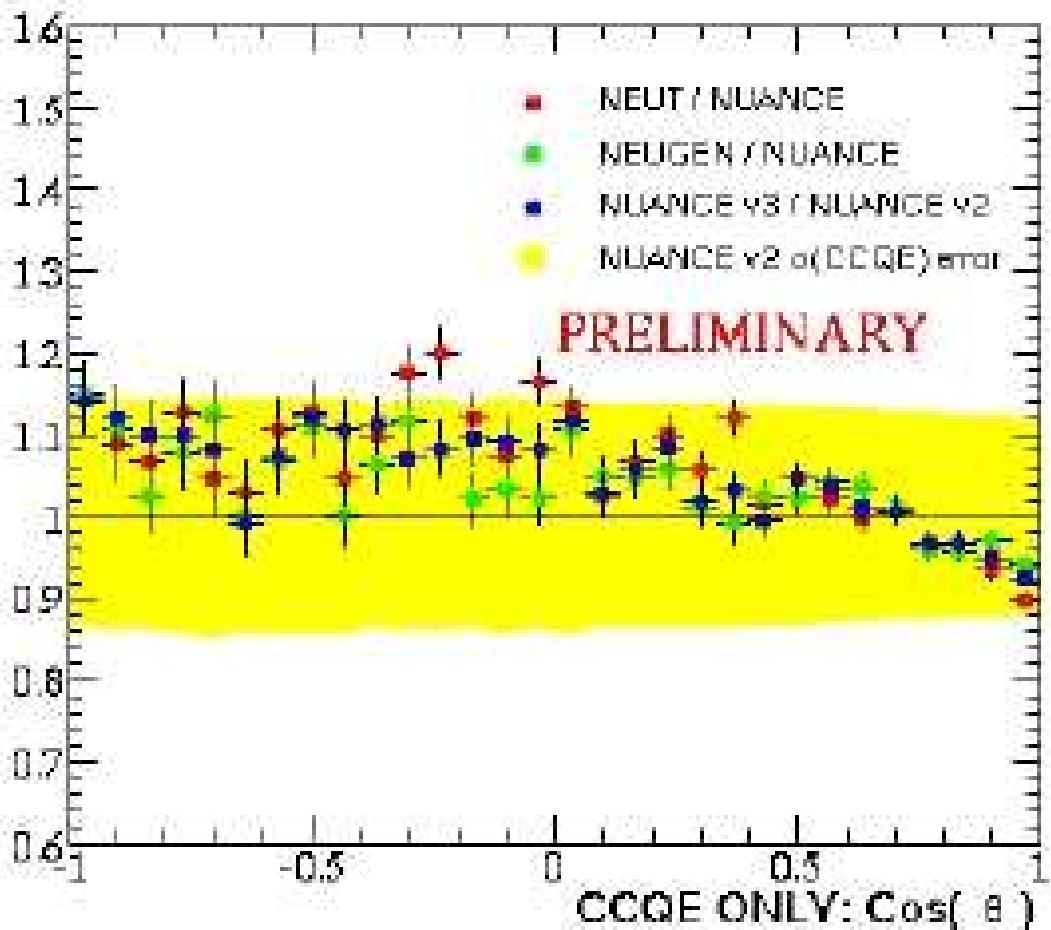
Dipole Form Factors, $M_A = 1.032\text{GeV}$

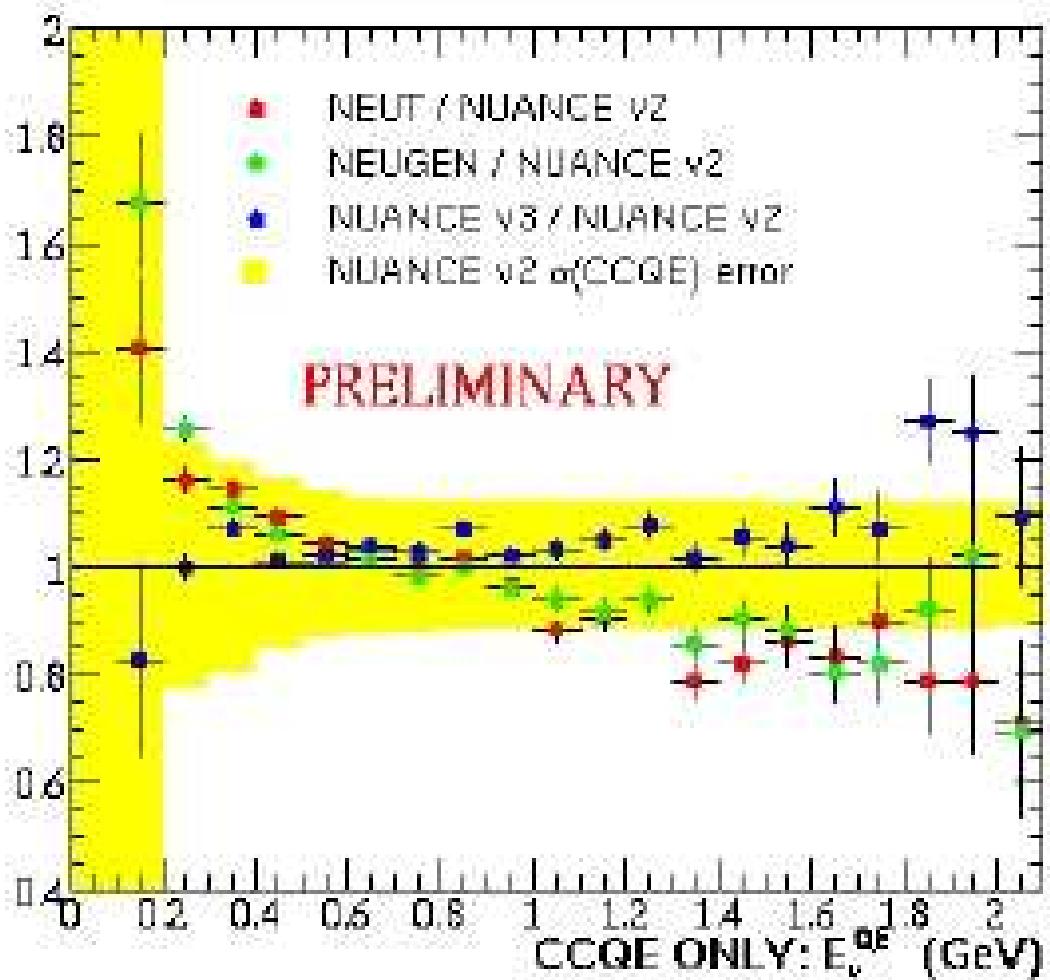
- Relativistic Fermi gas model of Gaisser and O'Connell

NEUT

Dipole Form Factors, $M_A = 1.1\text{GeV}$

- Fermi gas model of Smith and Moniz





→ Nuclear Effects are Important

NuInt - Workshops

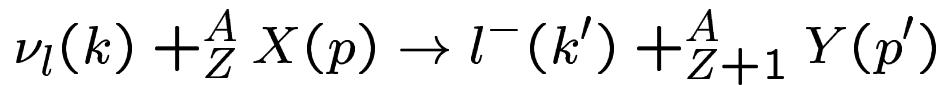
- Q. E. Scattering
- Resonance Reaction
- D. I. S.

Not Included Satisfactorily in Monte Carlo Event Generators

Nuclear Effects should be properly incorporated in Neutrino Event Generators for predicting Event Rates

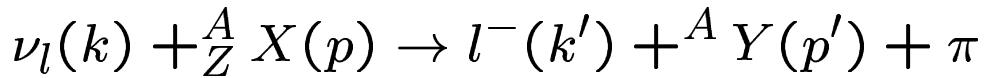
Nuclear Effects in the following processes

- Charged Current Quasielastic Scattering

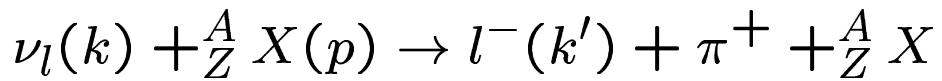


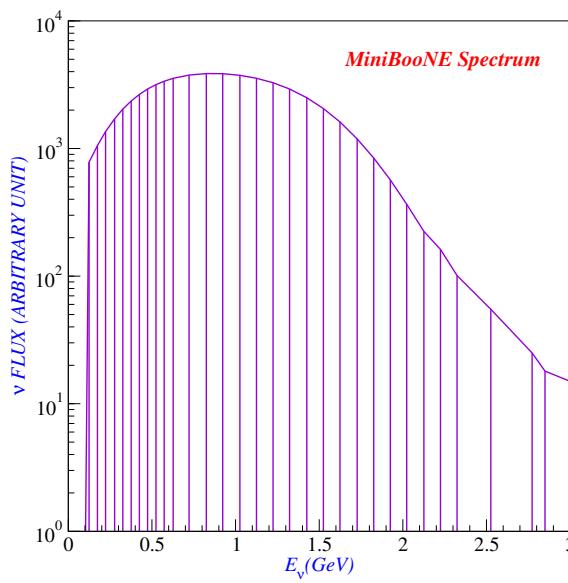
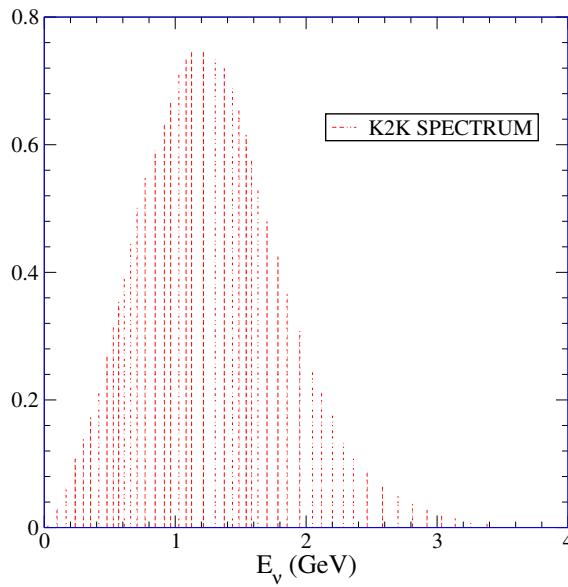
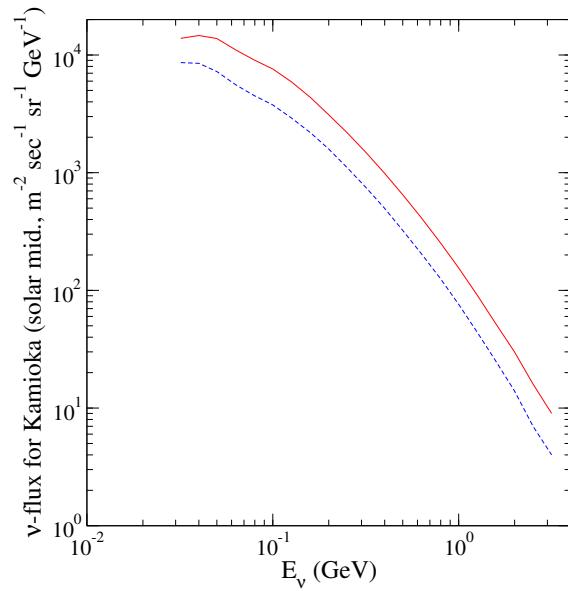
- Weak Pion Production

Incoherent Pion Production



Coherent Pion Production





Quasielastic Charged Current Reaction

The basic ν_e -neutron reaction taking place in ${}^A X$ nucleus is

$$\nu_l(k) + n(p) \rightarrow l^-(k') + p(p')$$

Matrix Element

$$T = \frac{G_F}{\sqrt{2}} \cos \theta_c \ l_\mu \ J^\mu$$

$$l_\mu = \bar{u}(k') \gamma_\mu (1 - \gamma_5) u(k)$$

$$\begin{aligned} J^\mu = & \bar{u}(p') [F_1^V(q^2) \gamma^\mu \\ & + F_2^V(q^2) i \sigma^{\mu\nu} \frac{q_\nu}{2M} + F_A^V(q^2) \gamma^\mu \gamma_5] u(p). \end{aligned}$$

$F_1^V(q^2)$, $F_2^V(q^2)$ and $F_A^V(q^2)$

are isovector form factors.

$F_1^V(0) = 1.0$, $F_2^V(0) = 3.7059$, Dipole mass $M_v = 0.84 \text{GeV}$, $M_A = 1.05 \text{GeV}$; $F_A(0) = -1.26$.

In a nucleus, the neutrino scatters from a neutron moving in the finite nucleus of neutron density $\rho_n(r)$, with a local occupation number $n_n(\mathbf{p}, \mathbf{r})$.

Scattering Cross Section in LDA

$$\left[\frac{d\sigma}{d\Omega_e dE_e} \right]_{Nucleus} = \int \rho_n(r) d^3r \left[\frac{d\sigma}{d\Omega_e dE_e} \right]_{FreeNucleon}$$

$$\rho_n(r) = 2 \int d\mathbf{p}_n \frac{1}{(2\pi)^3} n_n(\mathbf{p}, \mathbf{r})$$

$$E_n \rightarrow E_n(|\vec{p}|)$$

$$E_p \rightarrow E_p(|\vec{p} + \vec{q}|)$$

$$p_n < k_{F_n} \text{ and}$$

$$p'_p (= |\mathbf{p}_n + \mathbf{q}|) > k_{F_p},$$

$$k_{F_n} = [3\pi^2 \rho_n(r)]^{\frac{1}{3}}$$

$$k_{F_p} = [3\pi^2 \rho_p(r)]^{\frac{1}{3}}$$

To incorporate these modifications, the δ function $\delta[q_0 + E_n - E_p]$ is modified to $\delta[q_0 + E_n(\vec{p}) - E_p(\vec{p} + \vec{q}) - Q]$ and

$$\int \frac{d\mathbf{p}}{(2\pi)^3} n_n(\mathbf{p}, \mathbf{r}) \frac{M_n M_p}{E_n E_p} \delta[q_0 + E_n - E_p]$$

is replaced by $-(1/\pi) \text{Im} U_N(q_0, \vec{q})$,

$U_N(q_0, \vec{q})$ is the Lindhard function corresponding to the particle hole(ph) excitation

$$\begin{aligned} \sigma(E_\nu) = & -\frac{2G_F^2 \cos^2 \theta_c}{\pi} \int_{r_{min}}^{r_{max}} r^2 dr \int_{p_l^{min}}^{p_l^{max}} p_l^2 dp_l \times \\ & \int_{-1}^1 d(\cos\theta) \frac{1}{E_{\nu_l} E_l} L_{\mu\nu} J^{\mu\nu} \times \\ & \text{Im} U_N[E_{\nu_l} - E_l - Q, \vec{q}]. \end{aligned}$$

In the nucleus the strength of the electroweak coupling may change from their free nucleon values due to the presence of strongly interacting nucleons.

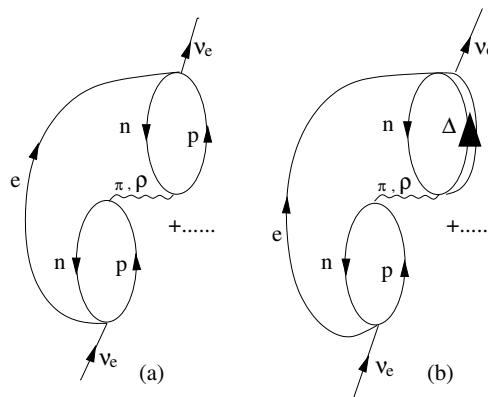
C.V.C. forbids any change in the charge coupling

Magnetic and Axial Vector Couplings are likely to change from their free nucleon values.

Quenching of Axial Current is a well established phenomena

These changes are calculated by considering the interaction of ph excitations in the nuclear medium in Random Phase Approximation (RPA).

The ph-ph interaction is shown by the wavy line and is described by the π and ρ exchanges modulated by the effect of short range correlations.



Many body Feynman diagrams (drawn in the limit $M_W \rightarrow \infty$) accounting for the medium polarization effects

The weak nucleon vertex described, in nonrelativistic limit, have terms like $F_A \vec{\sigma} \tau_+$ and $iF_2 \frac{\vec{\sigma} \times \vec{q}}{2M} \tau_+$ which generate spin-isospin transitions in nuclei.

These channels produce different RPA responses in the longitudinal and transvers channels when the diagrams are summed over.

$iF_2 \frac{\vec{\sigma} \times \vec{q}}{2M} \tau_+$ couples to the transverse excitations, the term $F_A \vec{\sigma} \tau_+$ couples to the transverse as well as longitudinal channels.

$$F_A^2 \delta_{ij} \text{Im} U_N \rightarrow F_A^2 [\hat{q}_i \hat{q}_j + (\delta_{ij} - \hat{q}_i \hat{q}_j)]$$

The RPA response of this term after summing the higher order diagrams gives a term which now modified as

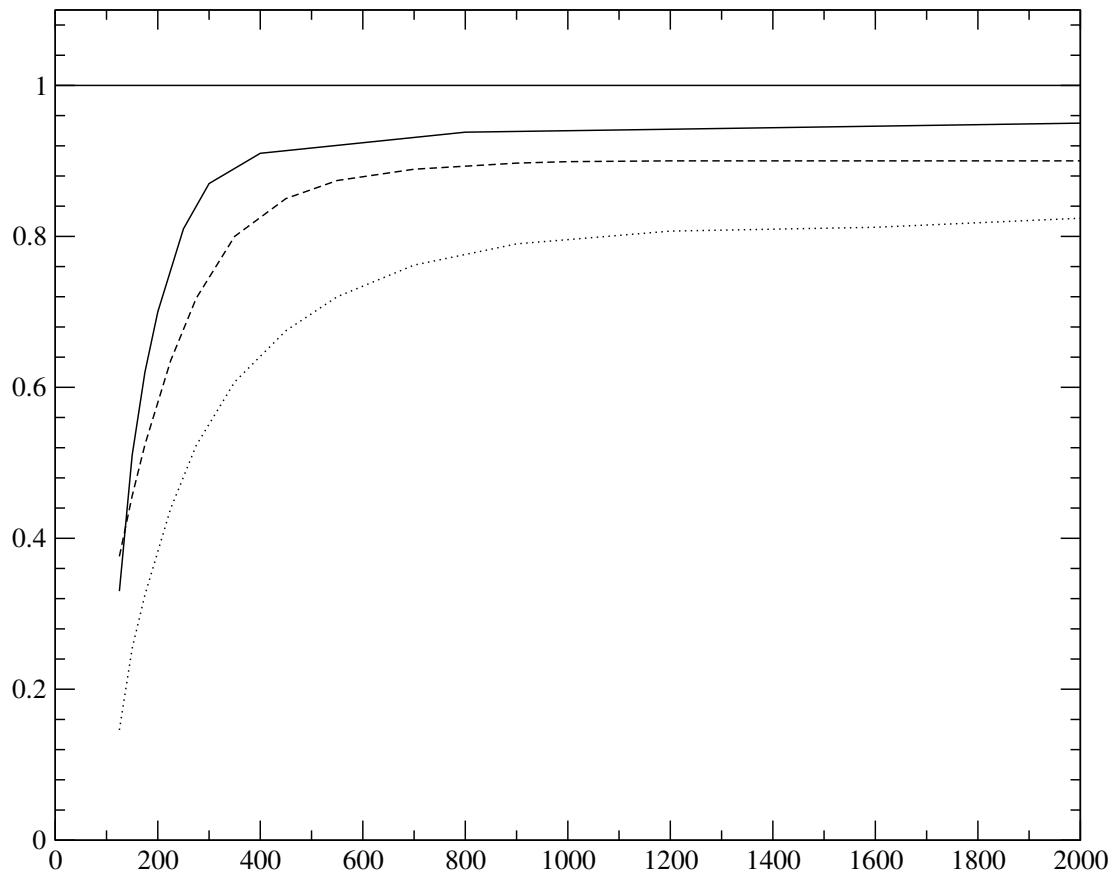
$$J^{ij} \rightarrow J_{RPA}^{ij} = J^{ij} F_A^2 U_N \left[\frac{\hat{q}_i \hat{q}_j}{1 - U_N V_l} + \frac{\delta_{ij} - \hat{q}_i \hat{q}_j}{1 - U_N V_t} \right]$$

where V_l and V_t are the longitudinal and transverse part of the nucleon-nucleon potential calculated with π and ρ exchanges.

$$V_l(q) = \frac{f^2}{m_\pi^2} \left[\frac{q^2}{-q^2 + m_\pi^2} \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q^2} \right)^2 + g' \right],$$

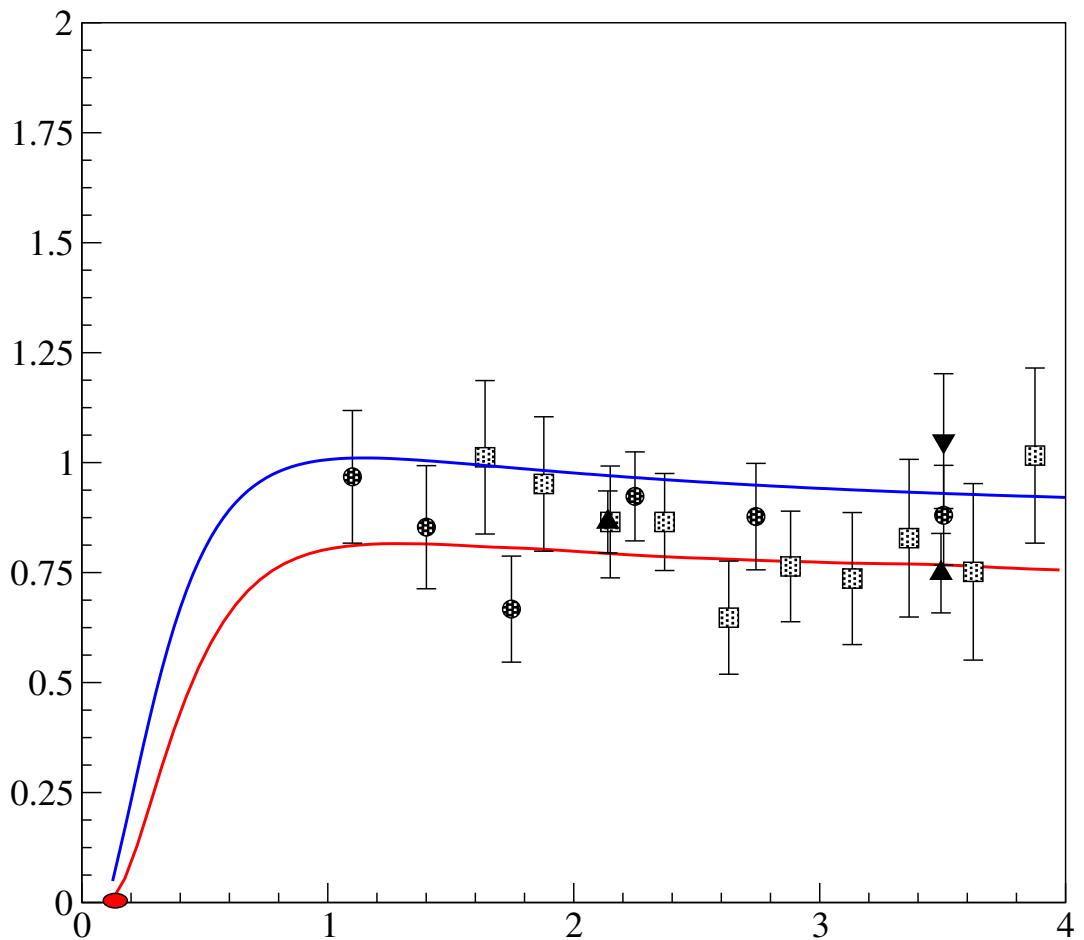
$$V_t(q) = \frac{f^2}{m_\pi^2} \left[\frac{q^2}{-q^2 + m_\rho^2} C_\rho \left(\frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - q^2} \right)^2 + g' \right]$$

$$\Lambda_\pi = 1.3 \text{GeV}, \quad C_\rho = 2, \quad \Lambda_\rho = 2.5 \text{GeV}.$$

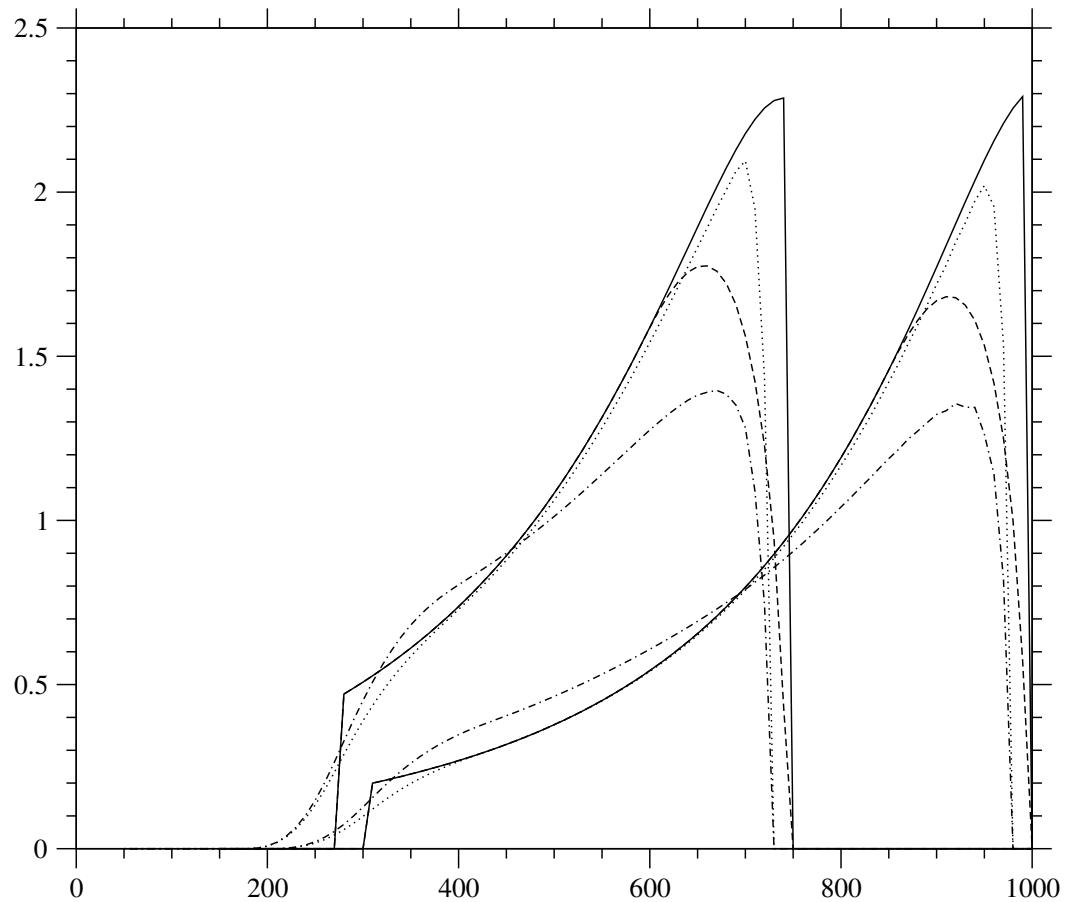


Ratio of the total cross section to the free neutrino nucleon cross section for the reactions (a) $\nu_\mu + n \rightarrow \mu^- + p$ in iron nuclei. without RPA (solid line), with RPA (dotted line) and Fermi gas model (dashed line)

E_ν (MeV)	FGM	with RPA
300	25	45
500	14	30
1000	10	20
1500	10	18
2000	10	17



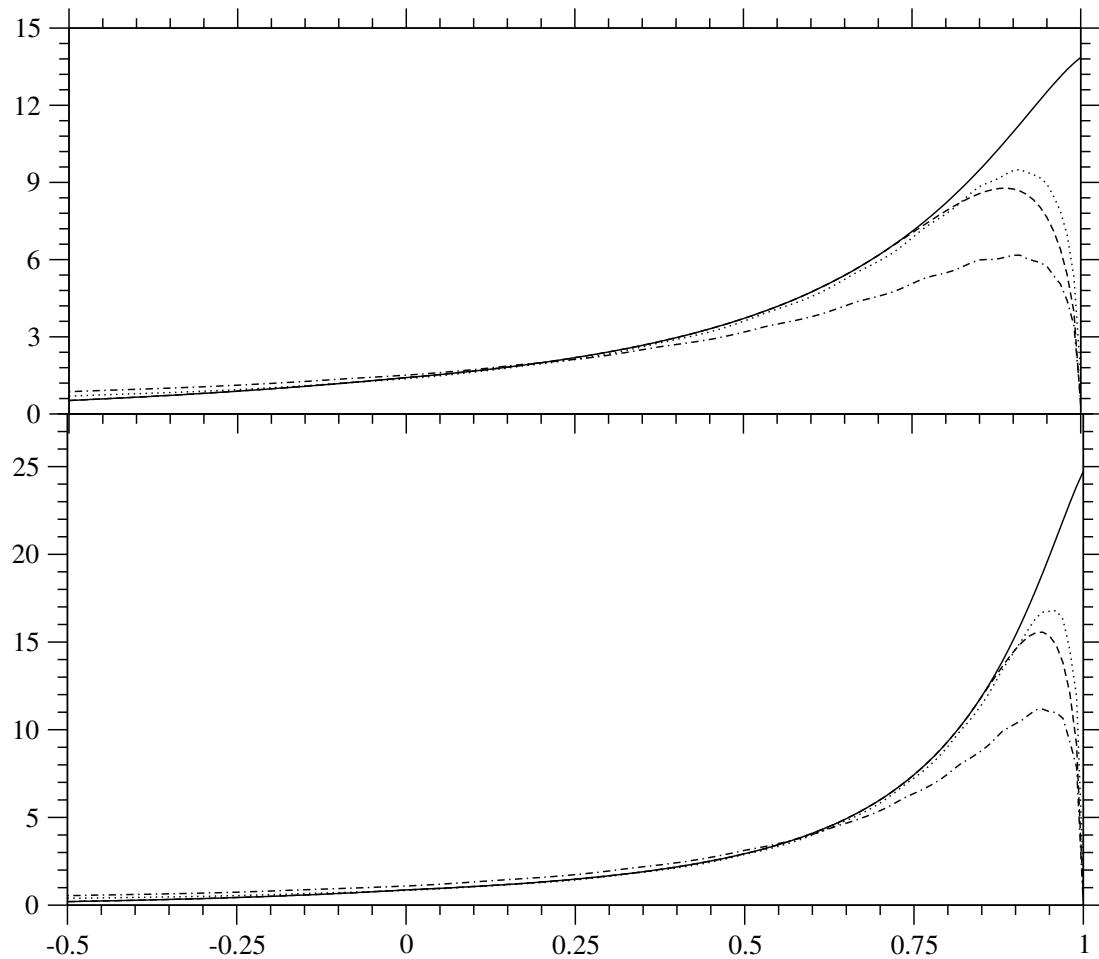
Neutrino quasielastic total cross section per nucleon($10^{-38}cm^2$) for $\nu_\mu + n \rightarrow p + \mu^-$ reaction in ^{56}Fe . The data are from LSND(Ellipse), Bonnelli et al.(squares), SKAT collab.(Triangle Down), Pohl et al.(Circle) and Belikov et al.(Triangle Up).



$\frac{d\sigma}{dp_l} (10^{-40} \frac{cm^2}{MeV})$ vs $p_l (MeV)$ at $E = 0.75 GeV$ and $E = 1.0 GeV$. without RPA(dotted line), with RPA(dashed-dotted line), free case(solid line) and FGM (dashed line).

% reduction in the differential cross section $\frac{d\sigma}{dp_l}$
 $E_\nu = 0.75 GeV$

$p_l (MeV)$	without RPA	with RPA (further reduction)
600	3	18
720	36	50
$E_\nu = 1.0 GeV$		
900	3	23
960	10	40



$\frac{d\sigma}{dcos\theta_l} (10^{-38} cm^2)$ vs $cos\theta_l$ at $E = 0.75 GeV$ and $E = 1.0 GeV$ without RPA(dotted line), with RPA(dashed-dotted line), free case(solid line) and FGM (dashed line)

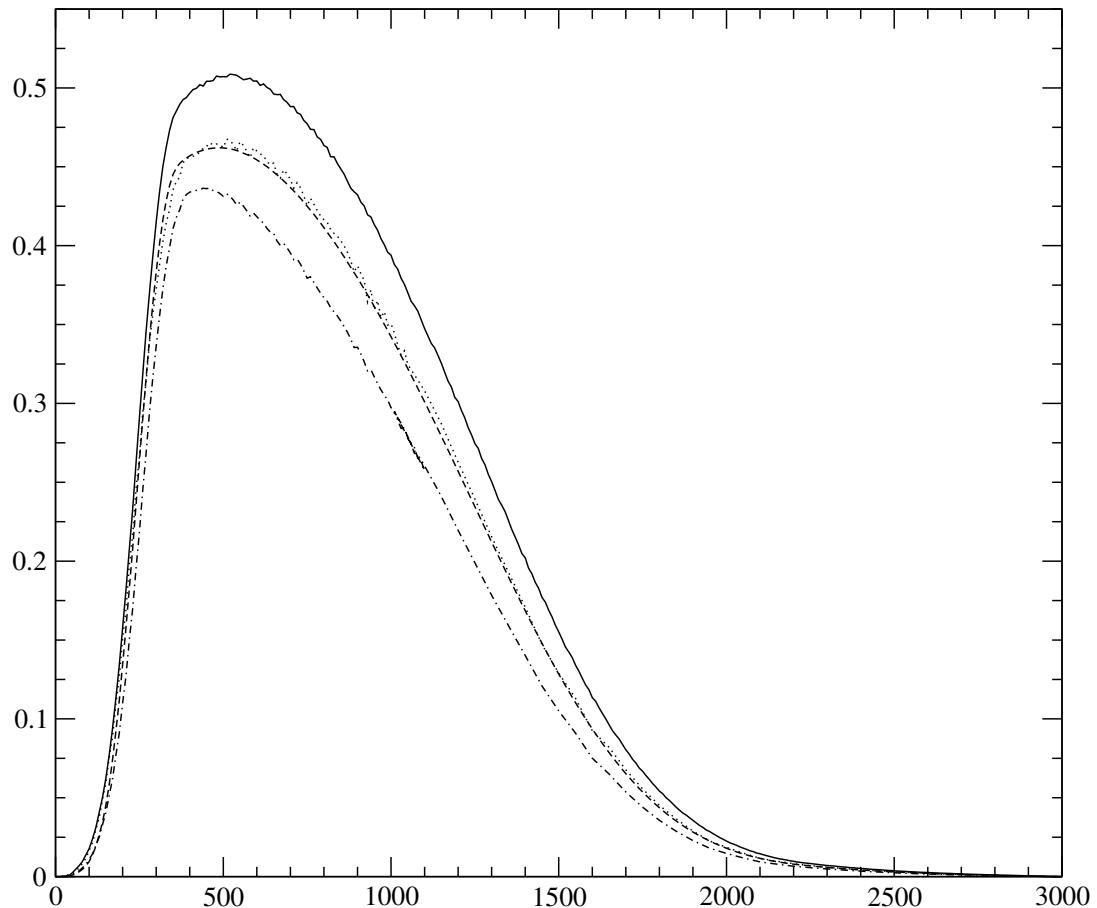
% reduction in the differential cross section $\frac{d\sigma}{dcos\theta_l}$

$E_\nu = 0.75 GeV$

$cos\theta_l(\theta_l)$	without RPA	with RPA (further reduction)
0.9(25)	14	35
0.996(5)	48	37

$E_\nu = 1.0 GeV$

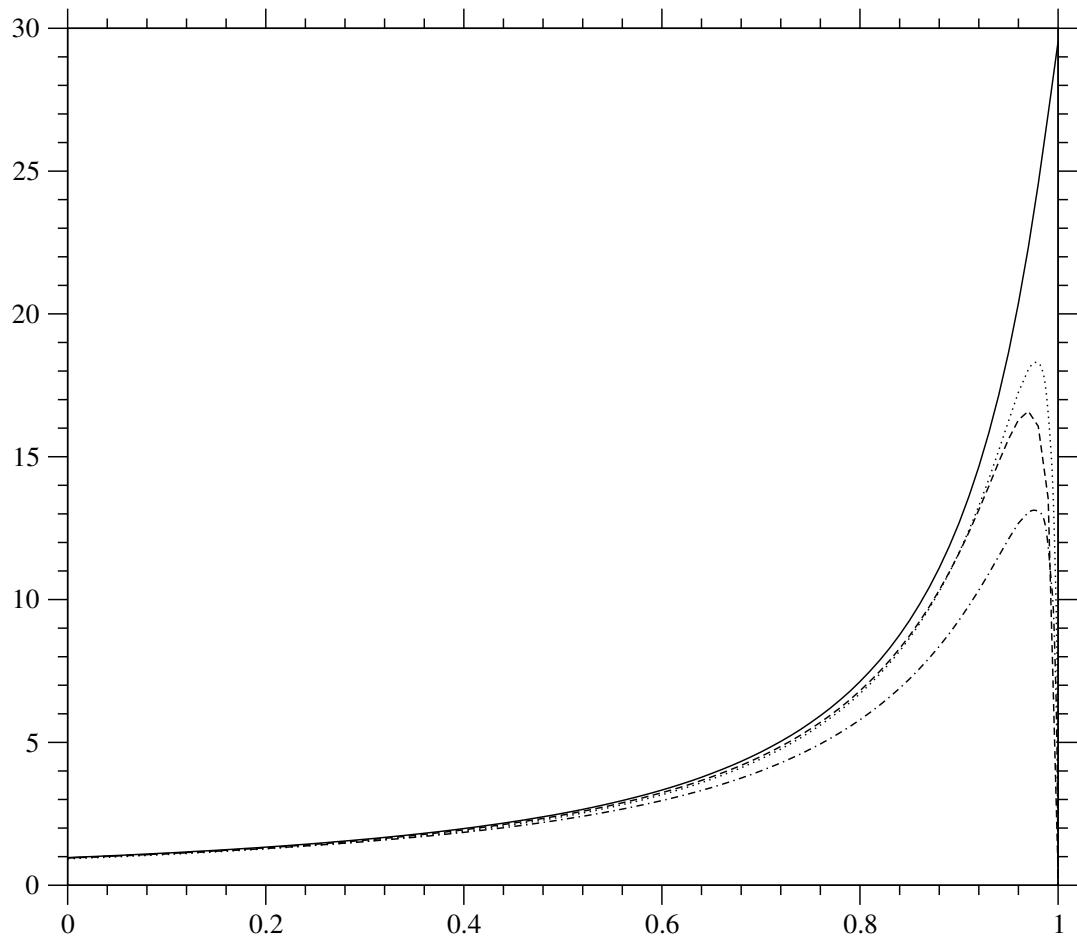
0.9(25)	5	30
0.996(5)	35	35



$\left\langle \frac{d\sigma}{dp_l} \right\rangle (10^{-40} \frac{cm^2}{MeV})$ vs p_l averaged over MiniBooNE spectrum. without RPA(dotted line), with RPA(dashed-dotted line), free case(solid line) and FGM (dashed line)

% reduction in the differential cross section $\left\langle \frac{d\sigma}{dp_l} \right\rangle$
 $p_l(MeV)$ without RPA with RPA
 (further reduction)

$p_l(MeV)$	without RPA	with RPA (further reduction)
500	9	7
800	10	12
1200	13	16
1500	16	18



$\frac{d\sigma}{dcos\theta_l} (10^{-38} cm^2)$ vs $cos\theta_l$ averaged over MiniBooNE spectrum. without RPA(dotted line), with RPA(dashed-dotted line), free case(solid line) and FGM (dashed line).

% reduction in the differential cross section

$$< \frac{d\sigma}{dcos\theta_l} >$$

$cos\theta_l(\theta_l)$	without RPA	with RPA (further reduction)
0.90(25)	8	20
0.996(5)	25	28

Inelastic Scattering Cross Section

In the intermediate energy region, the dominant contribution comes from the Δ resonance due to very strong P wave pion nucleon coupling leading to Δ resonance.

Matrix Element for the neutrino excitation through C. C.reaction

$$\begin{aligned}\nu_l(k) + p(p) &\rightarrow l^-(k') + \Delta^{++}(p') \\ \nu_l(k) + n(p) &\rightarrow l^-(k') + \Delta^+(p')\end{aligned}$$

is

$$T = \frac{G_F}{\sqrt{2}} \cos \theta_c \ l_\mu (V^\mu + A^\mu)$$

Matrix elements:

$$\begin{aligned}\langle \Delta^{++} | V^\mu | p \rangle &= \sqrt{3} \bar{\psi}_\alpha(p') \left(\frac{C_3^V(q^2)}{M} (g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) \right. \\ &+ \frac{C_4^V(q^2)}{M^2} (g^{\alpha\mu} q \cdot p' - q^\alpha p'^\mu) + \frac{C_5^V(q^2)}{M^2} (g^{\alpha\mu} q \cdot p - q^\alpha p^\mu) \\ &\left. + \frac{C_6^V(q^2)}{M^2} q^\alpha q^\mu \right) \gamma_5 u(p)\end{aligned}$$

$$\begin{aligned}\langle \Delta^{++} | A^\mu | p \rangle &= \sqrt{3} \bar{\psi}_\alpha(p') \left(\frac{C_3^A(q^2)}{M} (g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) \right. \\ &+ \frac{C_4^A(q^2)}{M^2} (g^{\alpha\mu} q \cdot p' - q^\alpha p'^\mu) \\ &\left. + C_5^A(q^2) g^{\alpha\mu} + \frac{C_6^A(q^2)}{M^2} q^\alpha q^\mu \right) u(p)\end{aligned}$$

$$C_5^V = 0, \quad C_4^V = -\frac{M}{M_\Delta} C_3^V, \quad \text{and}$$

$$C_3^V(q^2) = \frac{2.05}{(1 - \frac{q^2}{M_V^2})^2}, \quad M_V^2 = 0.54 \text{GeV}^2$$

The axial vector form factor $C_6^A(q^2)$ is related to $C_5^A(q^2)$ using PCAC and is given by

$$C_6^A(q^2) = C_5^A(q^2) \frac{M^2}{m_\pi^2 - q^2}$$

$$C_{i=3,4,5}^A(q^2) = C_i^A(0) \left[1 + \frac{a_i q^2}{b_i - q^2} \right] \left(1 - \frac{q^2}{M_A^2} \right)^{-2}$$

$$C_3^A(0) = 0, C_4^A(0) = -0.3, C_5^A(0) = 1.2, a_4 = a_5 = -1.21, \\ b_4 = b_5 = 2 \text{GeV}^2, M_A = 1.28 \text{GeV}.$$

Energy spectrum of the outgoing leptons

$$\frac{d^2\sigma}{dE_k d\Omega_{k'}} = \frac{1}{8\pi^3} \frac{1}{MM'} \frac{k'}{E_\nu} \frac{\frac{\Gamma(W)}{2}}{(W - M')^2 + \frac{\Gamma^2(W)}{4}} L_{\mu\nu} J^{\mu\nu}$$

where $W = \sqrt{(p+q)^2}$ and M' is mass of Δ ,

$$\begin{aligned} L_{\mu\nu} &= \bar{\Sigma} \Sigma l_\mu^\dagger l_\nu = L_{\mu\nu}^S + i L_{\mu\nu}^A \\ &= k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k \cdot k' + i \epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta \\ J^{\mu\nu} &= \bar{\Sigma} \Sigma J^{\mu\dagger} J^\nu \end{aligned}$$

$$P^{\mu\nu} = \sum_{spins} \psi^\mu \bar{\psi}^\nu$$

and is given by:

$$P^{\mu\nu} = -\frac{\not{p}' + M'}{2M'} \left(g^{\mu\nu} - \frac{2}{3} \frac{p'^\mu p'^\nu}{M'^2} + \frac{1}{3} \frac{p'^\mu \gamma^\nu - p'^\nu \gamma^\mu}{M'} - \frac{1}{3} \gamma^\mu \gamma^\nu \right)$$

The decay width Γ is taken to be an energy dependent P-wave decay width given by

$$\Gamma(W) = \frac{1}{6\pi} \left(\frac{f_{\pi N\Delta}}{m_\pi} \right)^2 \frac{M}{W} |\mathbf{q}_{cm}|^3 \Theta(W - M - m_\pi)$$

$$|\mathbf{q}_{cm}| = \frac{\sqrt{(W^2 - m_\pi^2 - M^2)^2 - 4m_\pi^2 M^2}}{2W}$$

In the nuclear medium the decay width Γ is modified

$$\tilde{\Gamma} = \frac{1}{6\pi} \left(\frac{f_{\pi N\Delta}}{m_\pi} \right)^2 \frac{M|\mathbf{q}_{cm}|^3}{W} F(k_F, E_\Delta, k_\Delta) \Theta(W - M - m_\pi)$$

$F(k_F, E_\Delta, k_\Delta)$ is the Pauli correction factor given by

$\Delta N \rightarrow NN$ and $\Delta NN \rightarrow NNN$

through which Δ 's disappear in the nuclear medium without producing a pion.

$M_\Delta \rightarrow M_\Delta + Re\Sigma_\Delta$ and $\tilde{\Gamma} \rightarrow \tilde{\Gamma} - Im\Sigma_\Delta$.

$$Re\Sigma_\Delta = 40 \frac{\rho}{\rho_0} MeV$$

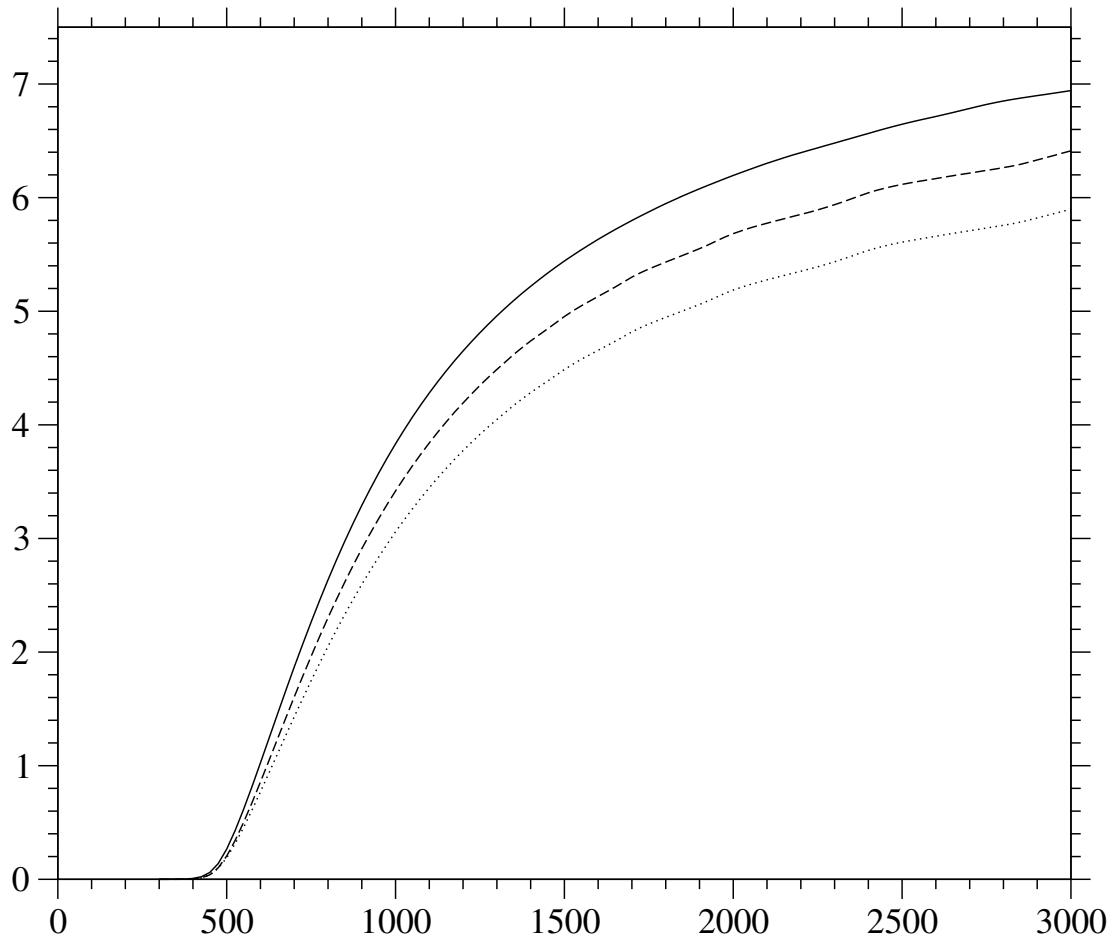
$$-Im\Sigma_\Delta = C_Q \left(\frac{\rho}{\rho_0} \right)^\alpha + C_{A2} \left(\frac{\rho}{\rho_0} \right)^\beta + C_{A3} \left(\frac{\rho}{\rho_0} \right)^\gamma$$

The cross section for π^+ production in the nucleus:

$$\sigma = \int \int \frac{d\mathbf{r}}{8\pi^3} \frac{d\mathbf{k}'}{E_\nu E_l} \frac{1}{MM'} \frac{\frac{\tilde{\Gamma}}{2} - Im\Sigma_\Delta}{(W - M' - Re\Sigma_\Delta)^2 + (\frac{\tilde{\Gamma}}{2} - Im\Sigma_\Delta)^2} \left[\rho_p(\mathbf{r}) + \frac{1}{3} \rho_n(\mathbf{r}) \right] L_{\mu\nu} J^{\mu\nu}$$

Expressions of $Re\Sigma_\Delta$ and $Im\Sigma_\Delta$: Oset et al. Nucl. Phys. A468 (1987) 631; Garcia Recio et al., Nucl. Phys. A526(1991)685

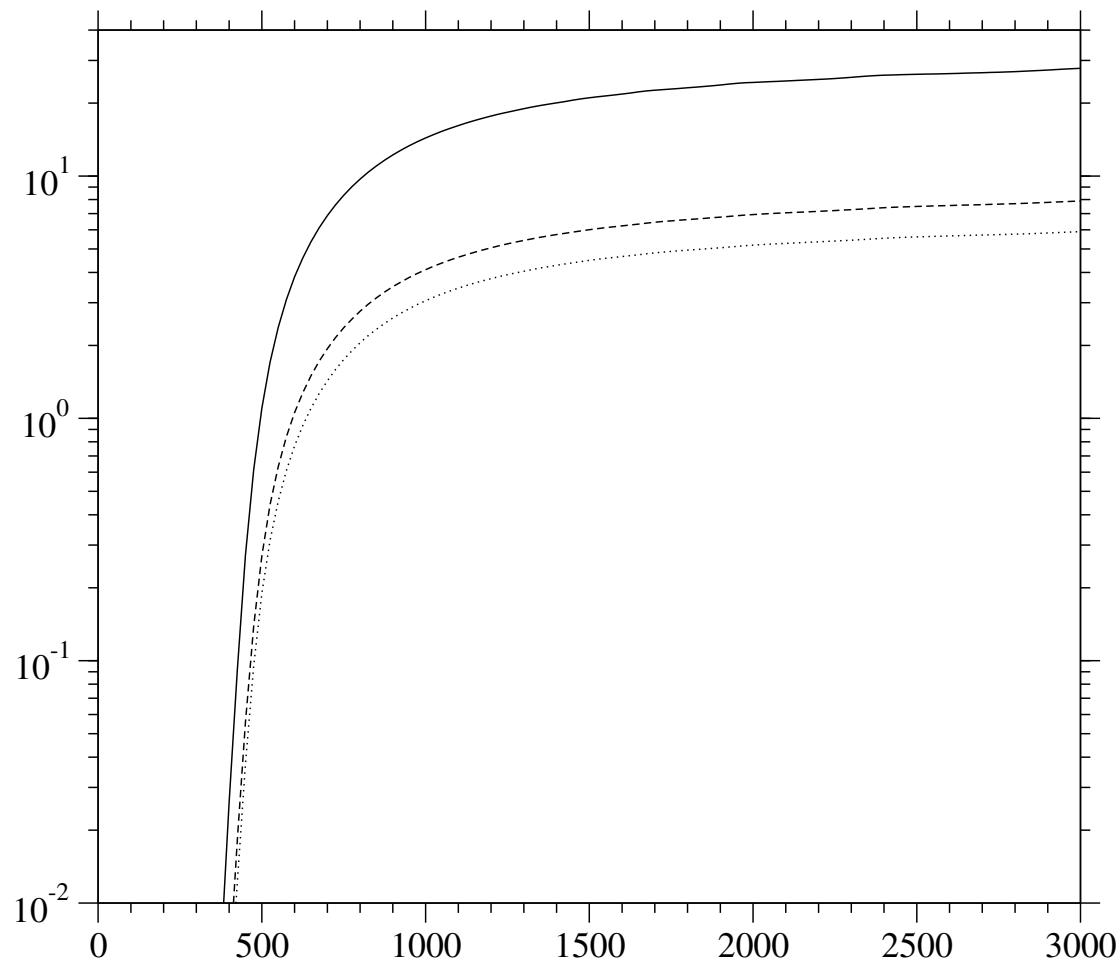
Pion Absorption Code Oset and Strottman, Phys. Rev. C42 (1990) 2454.



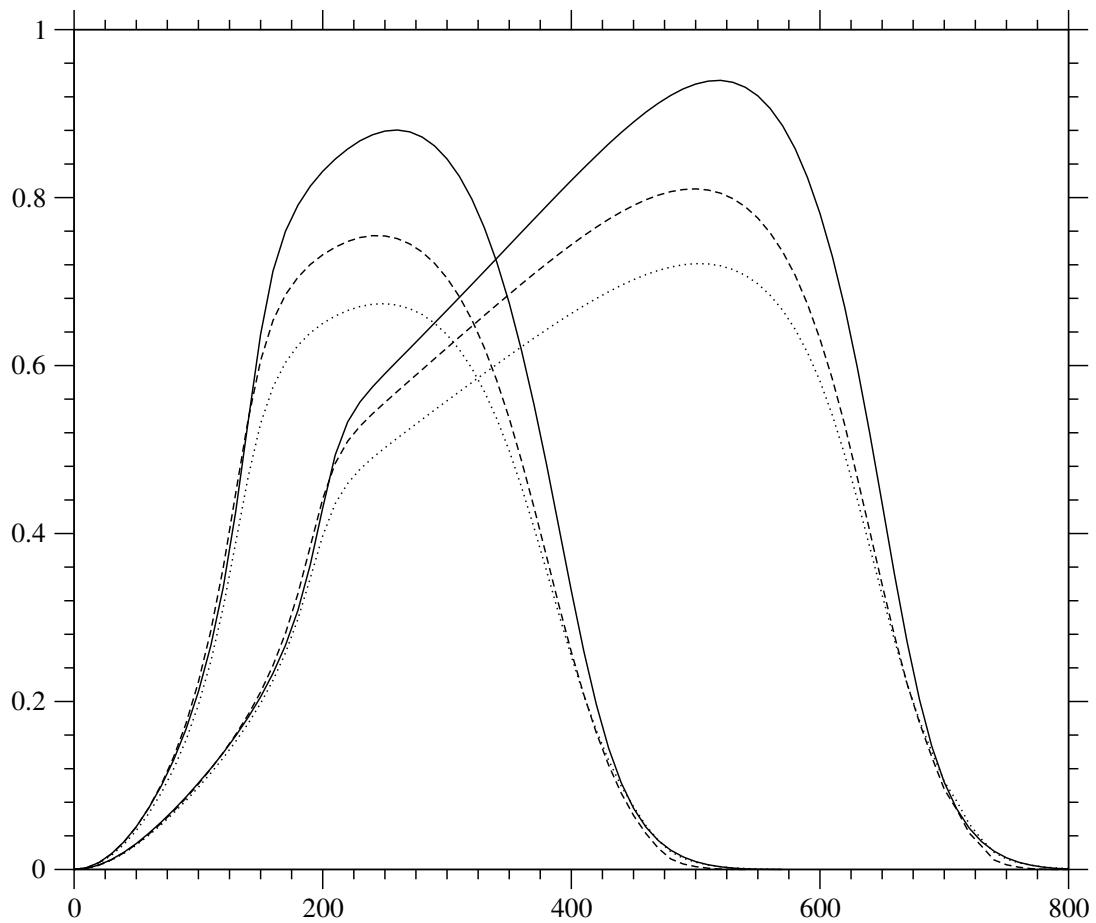
Total cross section (10^{-38} cm^2) for the reaction $\nu_l + N \rightarrow \Delta + l^-$ in ^{12}C without(solid line) and with nuclear medium effects(dashed line) and when pion absorption is also taken into account(dotted line).

% reduction in the total scattering cross section

σ	with NE	with NE and PA
500	23	8
1000	11	11
1500	9	10
3000	8	8



Total cross section $\sigma(10^{-38}cm^2)$ for the reaction $\nu_l + N \rightarrow \Delta + l^-$ in ^{12}C (dotted line), ^{16}O (dashed line) and ^{56}Fe (solid line) with nuclear medium and pion absorption effects.



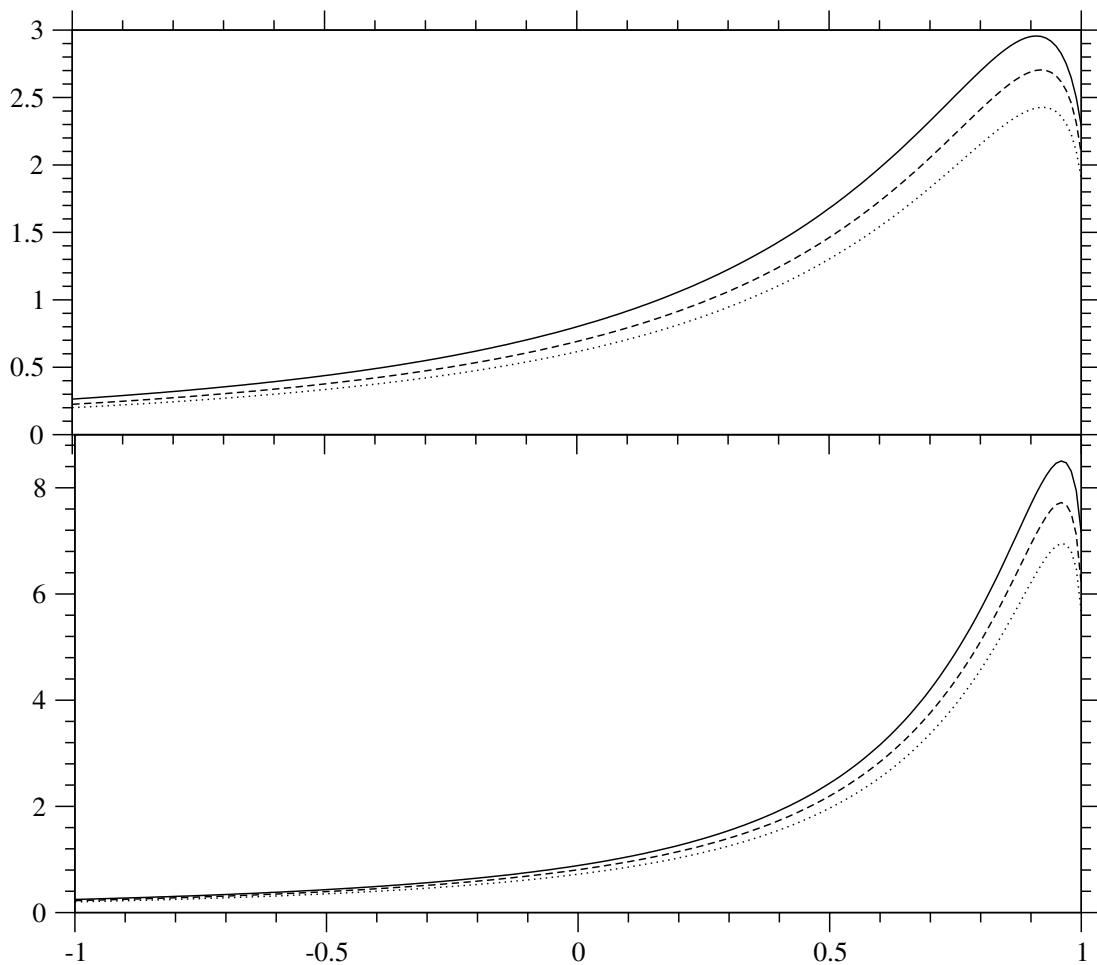
$\frac{d\sigma}{dp_l} (10^{-40} \frac{cm^2}{MeV})$ vs p_l at $E = 0.75 GeV$ and $E = 1.0 GeV$ for the reaction $\nu_l + N \rightarrow \Delta + l^-$ in ^{12}C without (solid line) and with nuclear medium effects (dashed line) and when pion absorption is also taken into account (dotted line).

% reduction in the differential cross section $\frac{d\sigma}{dp_l}$
 $E_\nu = 0.75 GeV$

$p_l (MeV)$	with NE	with NE and PA (further reduction)
200	15	11
300	17	10

$E_\nu = 1.0 GeV$

500	14	11
600	20	8



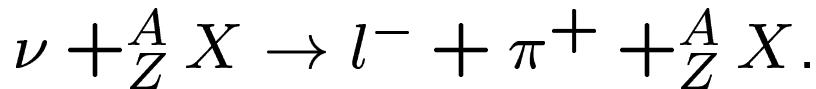
$\frac{d\sigma}{d\cos\theta_l}$ (10^{-38} cm^2 vs $\cos\theta_l$ at $E = 0.75 \text{ GeV}$ and $E = 1.0 \text{ GeV}$ for the reaction $\nu_l + N \rightarrow \Delta + l^-$ in ^{12}C without(solid line) and with nuclear medium effects(dashed line) and when pion absorption is also taken into account(dotted line)).

% reduction in the differential cross section $\frac{d\sigma}{d\cos\theta_l}$
 $\cos\theta_l(\theta_l)$ with NE with NE and PA
 (further reduction)

$E_\nu = 0.75 \text{ GeV}$			
$\cos\theta_l(\theta_l)$	with NE	with NE and PA	(further reduction)
0.9(25)	9	10	
0.996(5)	7	10	
$E_\nu = 1 \text{ GeV}$			
0.9(25)	9	10	
0.996(5)	9	10	

Coherent Weak Pion Production

The coherent pion production is the process in which the nucleus remains in the ground state. We calculate the coherent pion production induced by charged current interaction i.e.



The calculations are done in a local density approximation using Δ dominance:

Matrix Element

$$T = \frac{G_F}{\sqrt{2}} \bar{u}(k') \gamma^\mu (1 - \gamma_5) u(k) \\ (J_{direct}^\mu + J_{exchange}^\mu) F(\mathbf{q} - \mathbf{p}_\pi)$$

$$J_{direct}^\mu = \sqrt{3} \frac{G_F}{\sqrt{2}} \cos \theta_C \frac{f_{\pi N \Delta}}{m_\pi} t_\sigma^\pi \sum_s \bar{\Psi}^s(p') \Delta^{\sigma \lambda} O_{\lambda \mu} \Psi^s(p)$$

$$J_{exchange}^\mu = \sqrt{3} \frac{G_F}{\sqrt{2}} \cos \theta_C \frac{f_{\pi N \Delta}}{m_\pi} \sum_s \bar{\Psi}^s(p') t_\sigma^\pi O^{\sigma \lambda} \Delta_{\lambda \mu} \Psi^s(p)$$

$$F(\mathbf{q} - \mathbf{p}_\pi) = \int d^3r (\rho_p(\mathbf{r}) + \frac{1}{3} \rho_n(\mathbf{r})) e^{i(\mathbf{q} - \mathbf{p}_\pi) \cdot \mathbf{r}}$$

Using these expressions the following form of the double differential cross section for pion production is obtained

$$\frac{d^5\sigma}{d\Omega_\pi d\Omega_l dE_l} = \frac{1}{8} \frac{1}{(2\pi)^5} \frac{\mathbf{p}_1}{E_\nu} |\mathbf{p}_\pi| R \sum |T|^2$$

Pion Absorption

The approximate representation for the scattering wave function in Eikonal approximation

$$\psi(\mathbf{r}) = \exp \left[i \mathbf{p}_\pi \cdot \mathbf{r} - \frac{i}{v} \int_{-\infty}^z V(x, y, z') dz' \right]$$

Since the pion self energy $\Pi(\mathbf{r})$ is related to the equivalent pion optical potential $V(\mathbf{r})$ as

$$\Pi(\mathbf{r}) = 2E_\pi V(\mathbf{r})$$

$$\psi(\mathbf{r}) = \exp \left[i \mathbf{p}_\pi \cdot \mathbf{r} - i \int_{-\infty}^z \frac{\Pi(\mathbf{r}')}{2|\mathbf{p}_\pi|} dz' \right]$$

$$|\mathbf{v}| = |\mathbf{p}_\pi|/E_\pi.$$

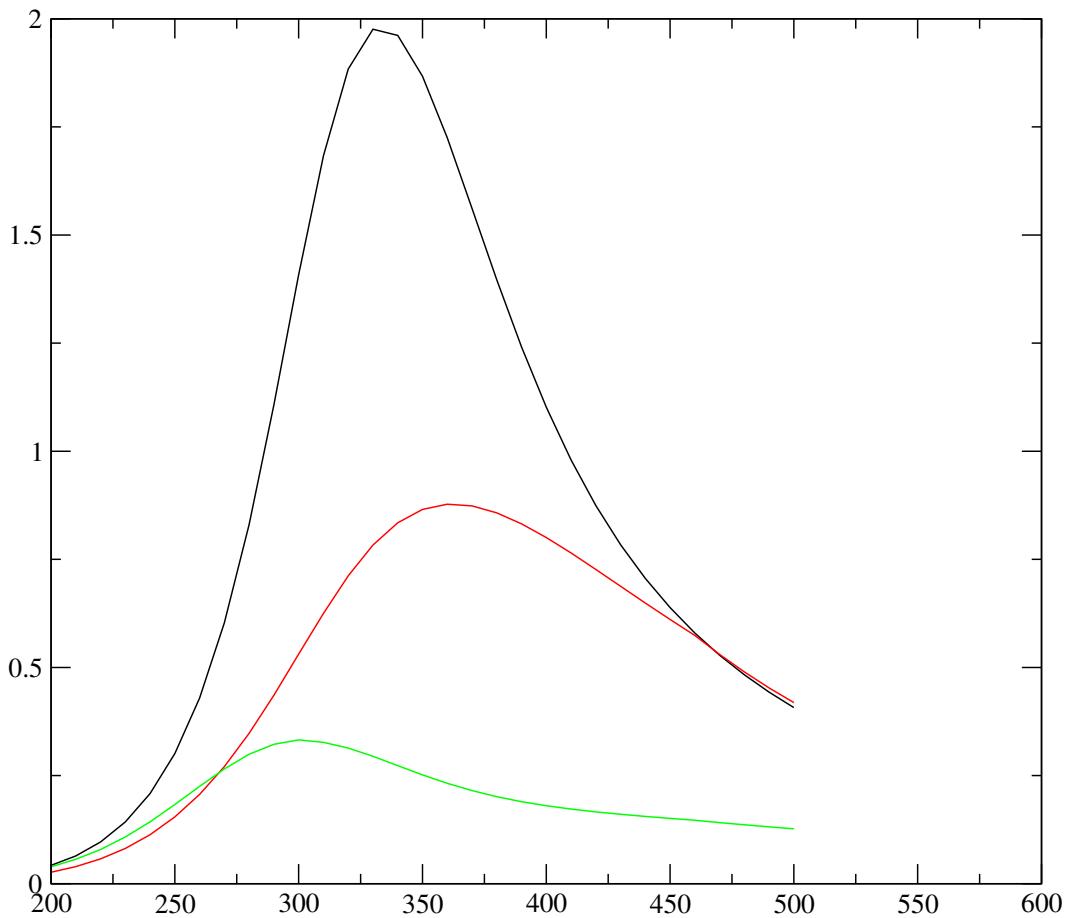
Using these expressions in impact parameter representation,

$$F(\mathbf{q} - \mathbf{k}_\pi) \quad \rightarrow \quad \tilde{F}(\mathbf{q} - \mathbf{k}_\pi)$$

$$\begin{aligned} \tilde{F}(\mathbf{q} - \mathbf{p}_\pi) &= \int d^2 b dz e^{-i(p_\pi^t \cdot b)} \\ &\times \exp \left[-i \int_z^\infty \frac{1}{2|\mathbf{p}_\pi|} \Pi(b, z') dz' \right] \\ &\times e^{i((q - p_\pi^l)z)} \rho(b, z) G_{\Delta h}(s, \rho(\mathbf{r})) \end{aligned}$$

$$p_\pi^l = \hat{\mathbf{q}} \cdot \mathbf{p}_\pi$$

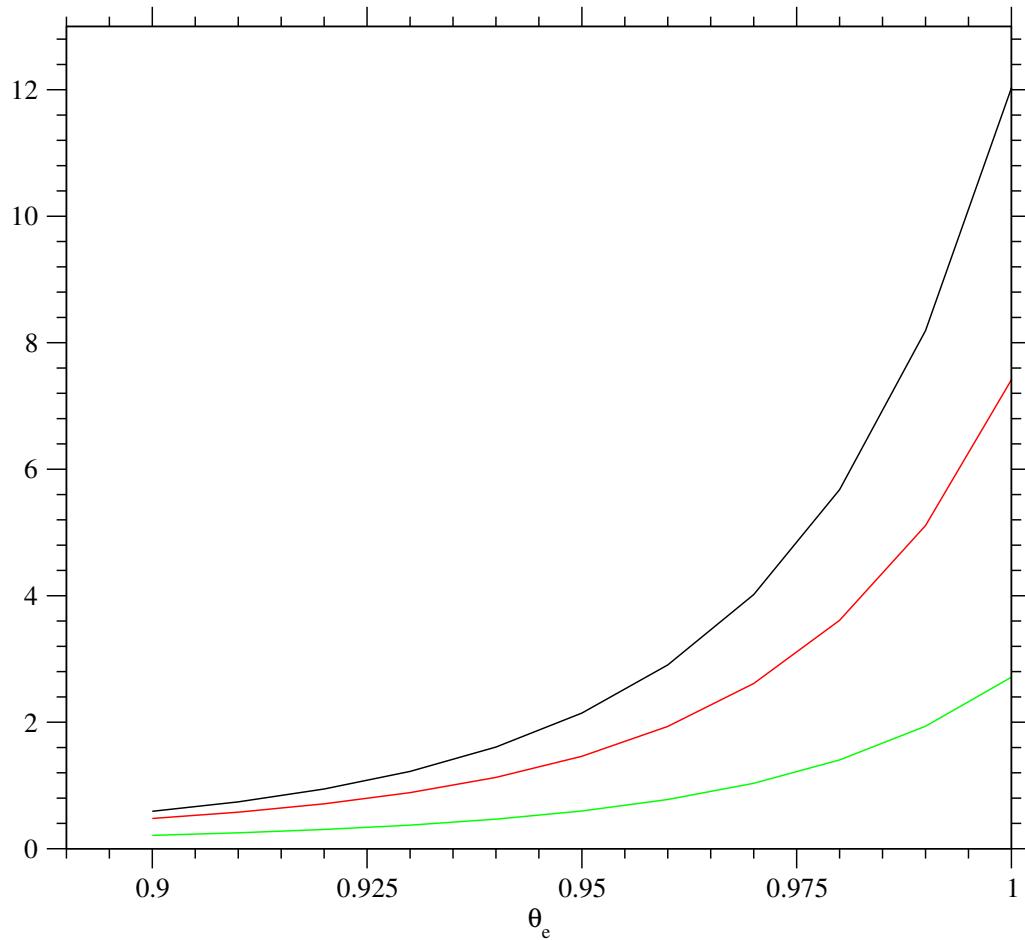
$$\begin{aligned} \tilde{F}(\mathbf{q} - \mathbf{p}_\pi) &= 2\pi \int b db dz J_0(p_\pi^t b) \\ &\times \exp \left[-i \int_z^\infty \frac{1}{2|\mathbf{p}_\pi|} \Pi(b, z') dz' \right] \\ &\times e^{i((q - p_\pi^l)z)} \rho(b, z) G_{\Delta h}(s, \rho(\mathbf{r})) \end{aligned}$$



$\frac{d\sigma}{dE_l} (10^{-38} \frac{cm^2}{MeV})$ vs q_0 at $E = 1.0 GeV$ for the
 coherent reaction $\nu_l + ^{16}_8 O \rightarrow l^- + ^{16}_8 O + \pi^+$
 without(black line) and with nuclear medium
 effects(red line) and when pion absorption is also
 taken into account(red line).

% reduction in the differential cross section $\frac{d\sigma}{dE_l}$
 $E_\nu = 1.0 GeV$

$q_0 (MeV)$	without RPA	with RPA (further reduction)
300	62	37
400	27	77

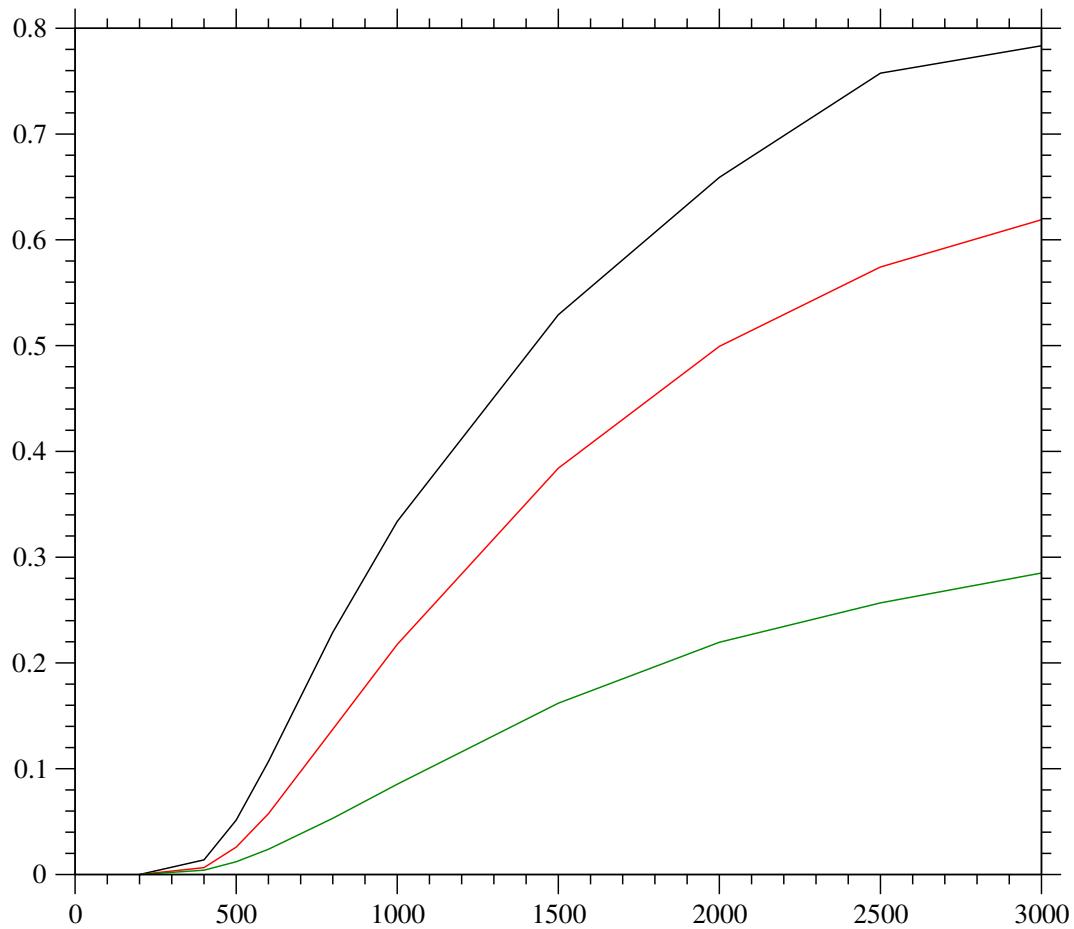


$\frac{d\sigma}{dcos\theta_l} (10^{-38} cm^2)$ vs $cos\theta_l$ at $E = 1.0 GeV$

% reduction in the differential cross section $\frac{d\sigma}{dcos\theta_l}$ for the coherent reaction $\nu_l + {}^{16}_8 O \rightarrow l^- + {}^{16}_8 O + \pi^+$

$E_\nu = 1.0 GeV$

$cos\theta_l(\theta_l)$	with PA	with PA (further reduction)
0.94(20)	30	58
1.00(0)	38	63

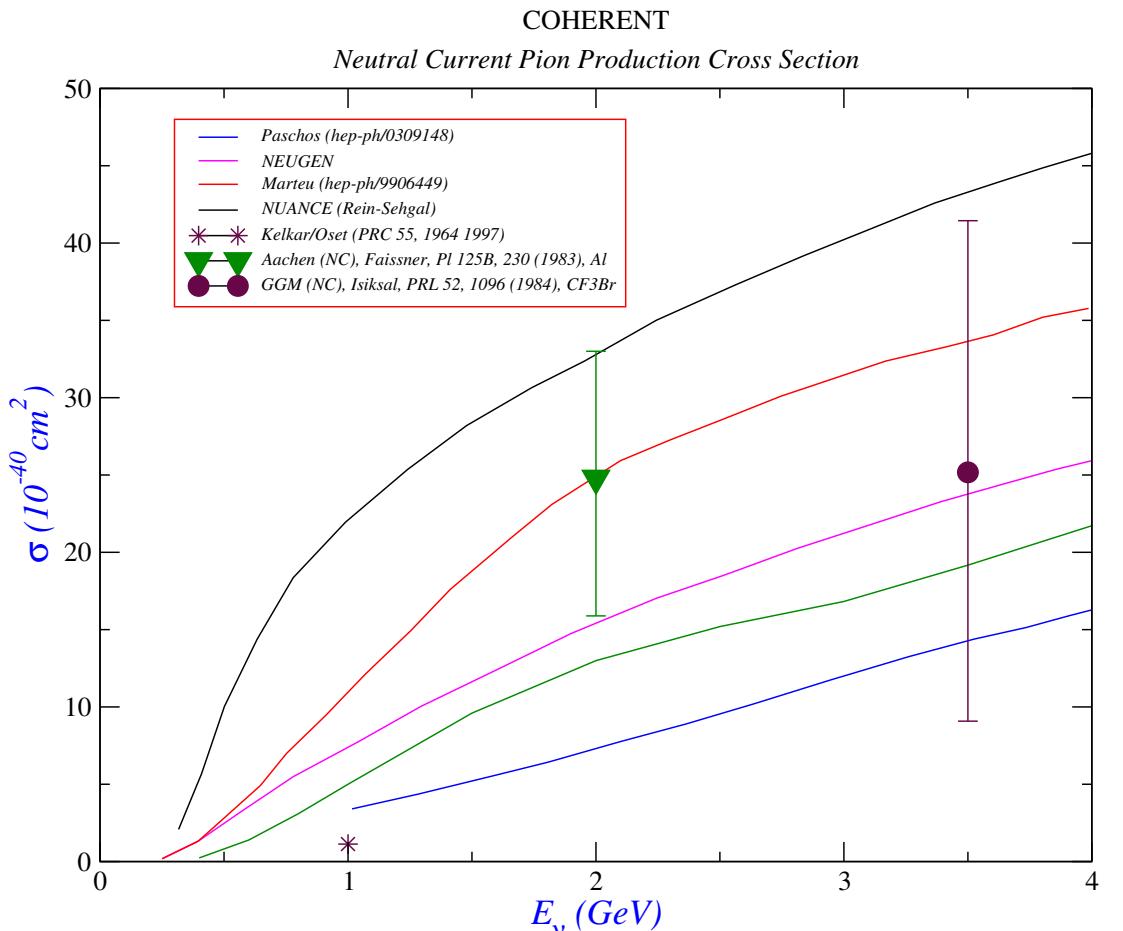


Total cross section(10^{-38} cm^2) at for the coherent reaction $\nu_l + {}^8_8 O \rightarrow l^- + {}^8_8 O + \pi^+$ without(black line) and with nuclear medium effects(red line) and when pion absorption is also taken into account(red line).

% reduction in the total scattering cross section

σ	with NE	with NE and PA (further reduction)
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σ	with NE	with NE and PA (further reduction)
600	46	58
1000	35	60
1500	27	58
2000	24	56



Coherent Neutral Current Pion Production in ^{16}O .