PH 802 Quantum Mechanics and Statistical Physics

Tutorial Sheet 2
(to be submitted on Thursday February 24, 2005)

---

1. Show that the following matrix can be diagonalized by a similarity transformation and obtain the orthogonal diagonalizing matrix

$$C = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2. The matrix

$$A = \begin{pmatrix} 1/2 & 0 & y \\ z & 1 & 0 \\ \sqrt{3}/2 & 0 & x \end{pmatrix}$$

is known to be orthogonal. For which of the sets of values obtained by you does A represent a rotation?

3. Two Hermitian operators anticommute

$$\{\hat{A}, \hat{B}\} = 0$$

Is it possible to have simultaneous eigenstates of these operators?

4. An operator $\hat{A}$ has two degenrate eigenstates with an eigenvalue a. If $\hat{B}$ is an operator such that $[\hat{A}, \hat{B}] = 0$, are these two states simultaneous eigenstates of $\hat{B}$. If yes prove it. If no, how would you construct eigenstates of $\hat{B}$.

5. Let

$$\delta_n(x) = \begin{array}{lrcl} 0, & x & < & -\frac{1}{2n} \\ n, & -\frac{1}{2n} & < x & < \frac{1}{2n} \\ 0, & \frac{1}{2n} & < & x \end{array}$$

Show that

$$lim_{n \to \infty} \int_{\infty}^{\infty} f(x)\delta_n(x)dx = f(0)$$

assuming that $f(x)$ is continuous at x = 0.

6. Prove

(i) $\delta(ax) = \frac{1}{a}\delta(x)$

(ii) $\delta(g(x)) = \sum_{ag(a)=0g'(a)\neq 0} \frac{\delta(x-a)}{|g'(a)|}$

7. For $\delta_n(x) = \frac{n}{\pi}\frac{1}{1+n^2\pi^2}$ show that

$$\int_{\infty}^{\infty} \delta_n(x)dx = 1$$

8. Demonstrate that $\delta_n(x) = \frac{sin(nx)}{\pi x}$ is a delta distribution by showing that

$$\int_{\infty}^{\infty} f(x)\frac{sin(nx)}{\pi x}dx = f(0)$$

9. Prove that

$$\delta(f(x)) = |\frac{df(x)}{dx}|^{-1}\delta(x - x_0)$$

10. Show that the expectation value of momentum for any well-behaved function is always real.

11. For an operator defined below, find out $\hat{P}|\psi\rangle$

$$\hat{P} = (\hat{X} + \frac{\partial}{\partial x})^2$$

12. There a large number (N) of identical experimental set-ups. In each of these set ups, a single particle is described by the wave function $\psi(x) = Ae^{-\frac{x^2}{a^2}}$, where A is the normalization constant, and a is a constant of the dimension length. If a measurement of position of the particle is carried out at time t=0 in all these set-ups, it is found that in 100 of these the particle is found within infinitesimal interval of x=2a and 2a+dx. Find out in how many of the measurements the particle would have been found in the infinitesimal interval of x=a and a+dx. Also find N.

13. $|\psi_1\rangle$ and $|\psi_2\rangle$ are the normalized eigen functions of an operator, with eigen values p_1 and p_2 respectively. If the wave function of the particle is at t=0; find the probability of observing p_1?

14. Consider a particle confined to a one-dimensional box. Find the probability that the particle in its ground state will be in the central one-third region of the box.

15. For a particle in one-dimensional box of side L, show that the probability of finding the particle between x = B and x = B + b approaches the classical value b/L, if the energy of the particle is very high.

16. Consider a one dimensional infinite square well potential of length L. A particle is in n=3 state of this potential well. Find the probability that this particle will be observed between x = 0 and x = (L/6). Can you guess the answer without solving the integral? Explain how.

17. For a particle in a one-dimensional box in nth state, show that the uncertainty product

$$\Delta X \Delta P_x = \hbar\sqrt{\frac{n^2\pi^2 - 6}{12}}$$

.

18. The wave function of a particle in a one-dimensional box of length L and potential zero at time t=0 is given as follows

$$\psi(x, 0) = \sqrt{\frac{2}{5L}}sin(\frac{\pi x}{L}) + \sqrt{\frac{8}{5L}}sin(\frac{4\pi x}{L})$$

   (a) Is this wave function normalized?

(b) Is the particle in a stationary state?

(c) If the energy measurement is made on his particle, what are the possible values one might get? What are the corresponding probabilities?

(d) What is the expectation value of the energy of this particle?

19. The wave function of a particle in a one-dimensional box of length L and potential zero at time t=0 is given as follows

$$\psi(x,0) = Asin(\frac{\pi x}{L}) + 2Asin(\frac{3\pi x}{L}) + \sqrt{11}Asin(\frac{5\pi x}{L})$$

(a) Find the wave function, at a later time t.

(b) If the measurements of energies are carried out, what are the values that will be found and what are the corresponding probabilities?

(c) If there are a large number of identical systems, each one of them represented by wave functions as above, what would be the average energy found if measurements are done in all of them at the same time

(d) If the measurement yielded the lowest value of energy, what would be the wave function later and what values of energy would be found later.

20. A particle in a one-dimensional box (V = 0 for $0 < x < L$, V $=\infty$ elsewhere) has the following wave function at t=0 between $0 < x < L$ and 0 elsewhere

$$\psi(x) = Asin(\frac{\pi x}{L})cos(\frac{2\pi x}{L})\text{between } 0 \ < \ \text{x} \ < \text{L and 0 elsewhere}$$

(a) Find A

(b) On making an energy measurement, which are the energy values that the particle may be found to have and with what probability?

21. A particle in a one-dimensional well (V=0 for $0 < x < L$, V $=\infty$ elsewhere) has the wave function $\psi(x) = Ax(L-x)$ inside the box and $\psi(x) = 0$ elsewhere at t=0. Calculate the expectation value of energy. On making an energy measurement, what is the probability of finding the particle in the ground state?