Neutrino mass hierarchy and $\theta_{13}$ with a magic baseline Beta Beam experiment

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The sign of $\Delta m^2_{31} \ (m^2_3 - m^2_1)$ is not known. Neutrino mass spectrum can be direct or inverted hierarchical.

Only an upper limit on $\sin^2 2\theta_{13} \ (< 0.17 \text{ at } 3\sigma)$ exists.

The CP phase ($\delta_{CP}$) is unconstrained.
The appearance probability $(\nu_e \rightarrow \nu_\mu)$ in matter, up to second order in the small parameters $\alpha \equiv \Delta m^2_{21}/\Delta m^2_{31}$ and $\sin 2\theta_{13}$, 

\[ P_{e\mu} \simeq \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2[(1 - \hat{A})\Delta]}{(1 - \hat{A})^2} \]

\[ + \alpha \sin 2\theta_{13} \xi \sin \delta \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})} \]

\[ + \alpha \sin 2\theta_{13} \xi \cos \delta \cos(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})} \]

\[ + \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}; \]

where $\Delta \equiv \Delta m^2_{31}L/(4E)$, $\xi \equiv \cos \theta_{13} \sin 2\theta_{21} \sin 2\theta_{23}$, 

and $\hat{A} \equiv \pm(2\sqrt{2}G_fn_eE)/\Delta m^2_{31}$.
Eight-fold Degeneracy

\[(\theta_{13}, \delta_{CP})\] intrinsic degeneracy

\[(\text{sgn}(\Delta m^2_{31}), \delta_{CP})\] degeneracy

\[(\theta_{23}, \pi/2 - \theta_{23})\] degeneracy

\[\Rightarrow\] severely deteriorates the sensitivity
If one chooses: \( \sin(\hat{A}\Delta) = 0 \)

- The \( \delta_{CP} \) dependence disappears from \( P_{e\mu} \)
- Golden channel enables a clean determination of \( \theta_{13} \) and \( sgn(\Delta m_{31}^2) \)

The first non-trivial solution: \( \sqrt{2}G_F n_e L = 2\pi \) (indep of E)

- Isoscalar medium of constant density \( \rho \):
  \[ L_{\text{magic}}[\text{km}] \approx 32725/\rho[\text{gm/cm}^3] \]
- According to PREM, the “magic baseline”

\[ L_{\text{magic}} = 7690 \text{ km} \]
The longer baseline captures a matter-induced contribution to the neutrino parameters, essential for probing the sign of $\Delta m_{31}^2$.

The CERN - INO baseline, close to the ‘magic’ value, ensures essentially no dependence of the final results on $\delta_{CP}$. This ‘magic’ value is independent of $E$.

This permits a clean measurement of $\theta_{13}$ avoiding the degeneracy issues which plague other baselines.
Resonance in matter effect

The very long CERN - INO baseline provides an excellent avenue to pin-down matter induced contributions.

In particular, a resonance occurs at

$$E_{\text{res}} \equiv \frac{|\Delta m^2_{31}| \cos 2\theta_{13}}{2\sqrt{2} G_F N_e}$$

\[= 6.1 \text{ GeV}\]

with \(|\Delta m^2_{31}| = 2.5 \times 10^{-3} \text{ eV}^2\), \(\sin^2 2\theta_{13} = 0.1\) and \(\rho_{av} = 4.13 \text{ gm/cc (PREM)}\) for the baseline of 7152 km.
Transition probability for different baselines

Normal vs. Inverted hierarchy

$\sin^2 2\theta_{13} = 0.1$ & all other osc. param. are fixed to their best-fit
Transition Probability $P_{e\mu}$

Transition probability for different baselines

Two different values of $\sin^2 2\theta_{13}$

Normal hierarchy & all other osc. param. are fixed to their best-fit

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What is Beta Beam?

- A pure, intense, collimated beam of $\nu_e$ or $\bar{\nu}_e$, essentially background free

- $\nu_e$ and $\bar{\nu}_e$ beams may also be produced at the same time in the set-up

- Origin: beta decay of completely ionized, radioactive ions circulating in a storage ring. No contamination of other types of neutrinos
Some positive features

- Known energy spectrum
- High intensity and low systematic errors
- High Lorentz boost of the parent ions ⇒ better collimation and higher energy of beam
- Can be produced with existing CERN facilities or planned upgrades
- Both $\nu_e$ and $\bar{\nu}_e$ beams can run simultaneously in the storage ring. The boost factors are fixed by their $e/m$ ratio
### Beta Beam: Ion sources

<table>
<thead>
<tr>
<th>Ion</th>
<th>$\tau$ (s)</th>
<th>$E_0$ (MeV)</th>
<th>$f$</th>
<th>Decay fraction</th>
<th>Beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{18}_{10}$Ne</td>
<td>2.41</td>
<td>3.92</td>
<td>820.37</td>
<td>92.1%</td>
<td>$\nu_e$</td>
</tr>
<tr>
<td>$^6_2$He</td>
<td>1.17</td>
<td>4.02</td>
<td>934.53</td>
<td>100%</td>
<td>$\bar{\nu}_e$</td>
</tr>
<tr>
<td>$^{8}_{5}$B</td>
<td>1.11</td>
<td>14.43</td>
<td>600684.26</td>
<td>100%</td>
<td>$\nu_e$</td>
</tr>
<tr>
<td>$^{8}_{3}$Li</td>
<td>1.20</td>
<td>13.47</td>
<td>425355.16</td>
<td>100%</td>
<td>$\bar{\nu}_e$</td>
</tr>
</tbody>
</table>

#### Comparison of different source ions

Low-$\gamma$ design, useful decays in case of anti-neutrinos can be $2.9 \times 10^{18}$/year and for neutrinos $1.1 \times 10^{18}$/year

Larger total end-point energy, $E_0$ is preferred
Boosted on-axis spectrum of $\nu_e$ and $\bar{\nu}_e$ at the far detector assuming no oscillation.
A magnetized Iron calorimeter (ICAL) detector with excellent efficiency of charge identification ($\sim 95\%$) and good energy determination

Preferred location is Singara (PUSHEP) in the Nilgiris (near Bangalore), 7152 km from CERN

A 50 kton Iron detector

Oscillation signal is the muon track ($\nu_e \rightarrow \nu_\mu$ channel)

Energy threshold is around 800 MeV
### Best-fit values

| $|\Delta m_{31}^2|$ | $2.5 \times 10^{-3}$ eV$^2$ |
|-----------------|--------------------------|
| $\sin^2 2\theta_{23}$ | 1.0                      |
| $\Delta m_{21}^2$  | $8.0 \times 10^{-5}$ eV$^2$ |
| $\sin^2 \theta_{12}$ | 0.31                     |
| $\delta_{CP}$    | 0                         |

Chosen benchmark values of oscillation parameters, except $\sin^2 2\theta_{13}$
### Detector assumptions

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Mass</td>
<td>50 kton</td>
</tr>
<tr>
<td>Energy threshold</td>
<td>1.5 GeV</td>
</tr>
<tr>
<td>Detection Efficiency ($\epsilon$)</td>
<td>60%</td>
</tr>
<tr>
<td>Charge Identification Efficiency ($f_{ID}$)</td>
<td>95%</td>
</tr>
</tbody>
</table>

#### Detector characteristics used in the simulations

We assume a Gaussian energy resolution function

with $\sigma = 0.15E$
Event Rates in INO-ICAL

Event rates

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Event Rates (contd.)

CERN − INO 8B Neutrino Beam
\( \gamma = 500 \) (NH)

Sensitivity to matter profile

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Minimum value of \( \sin^2 2\theta_{13}(\text{true}) \) as a function of \( \gamma \) at which the wrong hierarchy can be disfavored at the 90% and 3\( \sigma \) C.L. For \( \nu_e (\bar{\nu}_e) \) true hierarchy is assumed normal (inverted).

Marginalization over \( |\Delta m^2_{31}|, \sin^2 2\theta_{23}, \delta_{CP} \) and \( \sin^2 2\theta_{13} \)
Sensitivity to $\sin^2 2\theta_{13}$

$\sin^2 2\theta_{13}$ limit below which experiment is insensitive. For $\nu_e (\bar{\nu}_e)$ true hierarchy is assumed normal (inverted)

Marginalization over $|\Delta m^2_{31}|$, $\sin^2 2\theta_{23}$ and $\delta_{CP}$

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Conclusions

We have discussed the prospects of obtaining information on the mixing angle $\theta_{13}$ and the sign of $\Delta m^2_{31}$ using the proposed ICAL detector at INO with a Beta beam source at CERN.

The performance of the CERN - INO baseline is quite significant in comparison with other baselines avoiding the issue of degeneracy.

It appears that such a combination of a high intensity $\nu_e, \bar{\nu}_e$ source and a magnetized iron detector is well-suited for this purpose.