Large Matter Effects in $\nu_\mu \to \nu_\tau$ Oscillations

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<u>References</u>

hep-ph/0408361 (Phys. Rev. Lett. 94, 051801 (2005))

hep-ph/0411252 (submitted to Physical Review D.)

Motivation for Matter Effects

Matter effects arise due to the passage of neutrinos through the dense matter and the consequent modification of the propagation of different flavours because of the different forward scattering amplitudes.

Matter effects provide the energy dependence which is needed to solve solar neutrino problem. And they establish that $\Delta_{sol} = \Delta_{21}$ is positive.

Establishing matter effects in atmospheric neutrino oscillations (or long baseline neutrino oscillations) is crucial to determining the sign of $\Delta_{atm} = \Delta_{31}$.

Three Flavour Oscillations

To discuss matter effects in the context of atmospheric neutrino oscillations we necessarily have to consider three flavour oscillations.

We assume mixing of three neutrino flavours which give rise to three mass eigenstates. Two of the mass eigenstates are split by Δ_{sol}. We label the lower of these ν_1 and the higher by ν_2. ν_3, whose splitting from ν_1/ν_2 is given by Δ_{atm}, can be either above (positive Δ_{31}) or below (negative Δ_{31}) the two close mass eigenstates.

The mixing matrix is parametrized in a form advocated by Kuo-Pantaleone (RMP 61, 937 (1989))

$$
\begin{aligned}
\nu_{flavour} &= U \nu_{mass} \\
U &= U_{23}(\theta_{23}) U_{phase} U_{13}(\theta_{13}) U_{12}(\theta_{12}),
\end{aligned}
$$

where

$$
U_{12}(\theta_{12}) = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.
$$

U_{phase} is a diagonal matrix containing a CKM like phase.

One Mass Scale Dominance Approximation

Because $\Delta_{21} << \Delta_{31}$, we can set $\Delta_{21} = 0$ for appropriate energy and pathlength ranges.

Then θ_{12} and the phase also become irrelevant for neutrino oscillation probabilities.

Survival and oscillation probabilities depend only the three parameters Δ_{31}, θ_{23} and θ_{13}.

We take $\Delta_{31} = 2 \times 10^{-3}$ eV^2, $\theta_{23} = 45^\circ$ and $\theta_{13} \leq 15^\circ$.

All the algebraic equations are given for OMSD approximation but the graphs and numbers are obtained by full numerical calculation with non-zero values of Δ_{21} *and* θ_{12}.

Matter Effects in $\nu_\mu \to \nu_e$ Oscillations

It is expected that matter effects will be most visible in $\nu_\mu \to \nu_e$ oscillations because the origin of matter effects is the difference in the forward scattering amplitude off electrons of ν_e (due to both charged current and neutral current) and ν_μ/ν_τ (due to only neutral current).

$$P^{vac}(\nu_\mu \to \nu_e) = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2(1.27\Delta_{31}L/E),$$

which is the same for both positive and negative Δ_{31}.

Including matter effects changes this to

$$P^{mat}(\nu_\mu \to \nu_e) = \sin^2\theta_{23}\sin^2 2\theta_{13}^m \sin^2(1.27\Delta_{31}^m L/E),$$

where

$$\begin{aligned}
\sin^2 2\theta_{13}^m &= \sin^2 2\theta_{13}\Delta_{31}/\Delta_{31}^m \\
\Delta_{31}^m &= \sqrt{(\Delta_{31}\cos 2\theta_{13} - A)^2 + (\Delta_{31}\sin 2\theta_{13})^2} \\
A &= 0.76 \times 10^{-4}\rho(\text{in gm/cc})E(\text{in GeV})
\end{aligned}$$

Maximising Matter Effects in $\nu_\mu \to \nu_e$

From the expression for $P^{mat}(\nu_\mu \to \nu_e)$ we see that, for positive Δ_{31}, resonance occurs when ρ and E are such that $A = \Delta_{31} \cos 2\theta_{13}$ is satisfied.

At resonance we have $\sin^2 2\theta_{13}^m = 1$ and it seems as $P^{mat}(\nu_\mu \to \nu_e)$ is increased many fold.

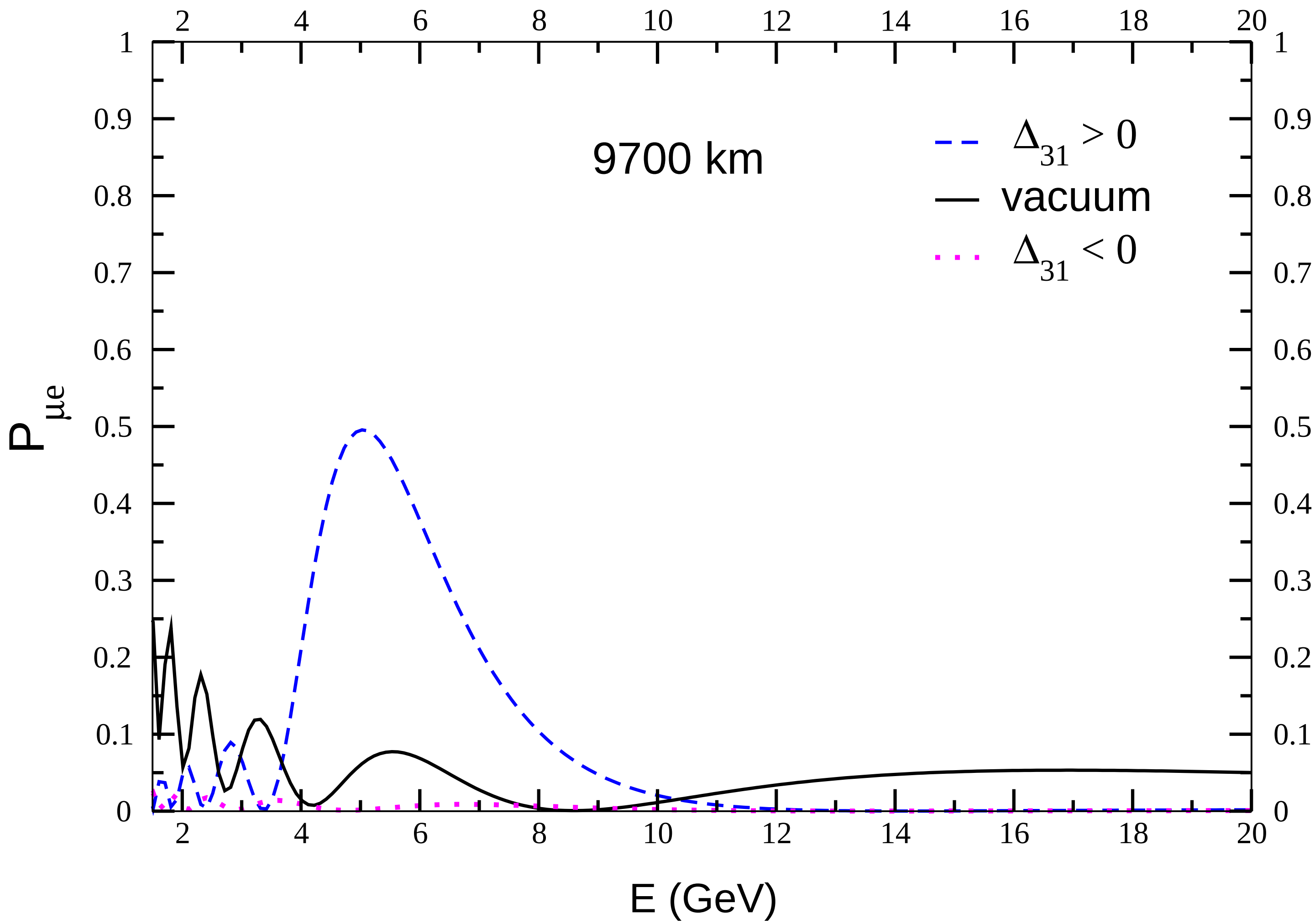
But at resonance $\Delta_{31}^m = \Delta_{31} \sin 2\theta_{13}$ and the oscillating factor becomes very small.

So, to maximise $P^{mat}(\nu_\mu \to \nu_e)$, we have to be close to the resonance energy and also choose a pathlegth such that the oscillating factor is maximised.

This gives us a condition on pathlength

$$\rho L_{\mu e}^{max} = \frac{\pi 5.18 \times 10^3}{\tan 2\theta_{13}}.$$
$$L_{\mu e} \simeq 10,000 \text{ Km}$$

for $\theta_{13} \simeq 10^\circ$.



9700 km
$\Delta_{31} > 0$
vacuum
$\Delta_{31} < 0$
$P_{\mu e}$
E (GeV)

Observing the resonance amplification of matter effects in $\nu_\mu \to \nu_e$ is all but impossible except for the largest possible values of θ_{13}.

For negative Δ_{31} no resonance occurs for neutrinos. Hence $P^{mat}(\nu_\mu \to \nu_e)$ will not differ very much from its vacuum value. It will actually be lower. Resonance occurs for anti-neutrinos.

Matter Effects in $\nu_\mu \to \nu_\tau$ Oscillations

$$\begin{aligned} \mathrm{P}_{\mu\tau}^{\mathrm{vac}} &= \cos^4\theta_{13}\sin^2 2\theta_{23}\sin^2(1.27\Delta_{31}\mathrm{L}/\mathrm{E}), \\ &= \cos^2\theta_{13}\sin^2 2\theta_{23}\sin^2(1.27\Delta_{31}L/E) \\ &\quad - \cos^2\theta_{23}\mathrm{P}_{\mu\mathrm{e}}^{\mathrm{vac}} \end{aligned}$$

Including matter effects we get

$$\begin{aligned} \mathrm{P}_{\mu\tau}^{\mathrm{mat}} &= \cos^2\theta_{13}^{\mathrm{m}}\sin^2 2\theta_{23}\sin^2[1.27(\Delta_{31}+\mathrm{A}+\Delta_{31}^{\mathrm{m}})\mathrm{L}/2\mathrm{E}] \\ &\quad \sin^2\theta_{13}^{\mathrm{m}}\sin^2 2\theta_{23}\sin^2[1.27(\Delta_{31}+\mathrm{A}-\Delta_{31}^{\mathrm{m}})\mathrm{L}/2\mathrm{E}] \\ &\quad - \cos^2\theta_{23}\mathrm{P}_{\mu\mathrm{e}}^{\mathrm{mat}} \end{aligned}$$

Lifting the $\nu_1 - \nu_2$ Degeneracy due to Matter Effects

This complicated dependence arises due to the following reason. Three flavour oscillation probabilities, in general, there are three oscillating terms one of which contains Δ_{21}, the second Δ_{31} and the third $\Delta_{32} = \Delta_{31} - \Delta_{21}$.

If Δ_{21} set to zero, the first term drops out and the second and third terms combine to form a single term. We see this in both $P(\nu_\mu \to \nu_e)$ and in $P(\nu_\mu \to \nu_\tau)$.

But when matter effects are included, the $\nu_1 - \nu_2$ degeneracy is broken and we have three different matter dependent mass-square differences. These are $(\Delta_{31} + A + \Delta_{31}^m)/2$, $(\Delta_{31} + A - \Delta_{31}^m)/2$ and Δ_{31}^m.

$P^{mat}(\nu_\mu \to \nu_e)$ is not affected by the breaking of this degeneracy because one of the previously degenerate mass eigenstates (ν_2) contains no ν_e component. Hence it does not contribute to $\nu_\mu \to \nu_e$ oscillations, which are driven by a single mass-square difference Δ_{31}^m.

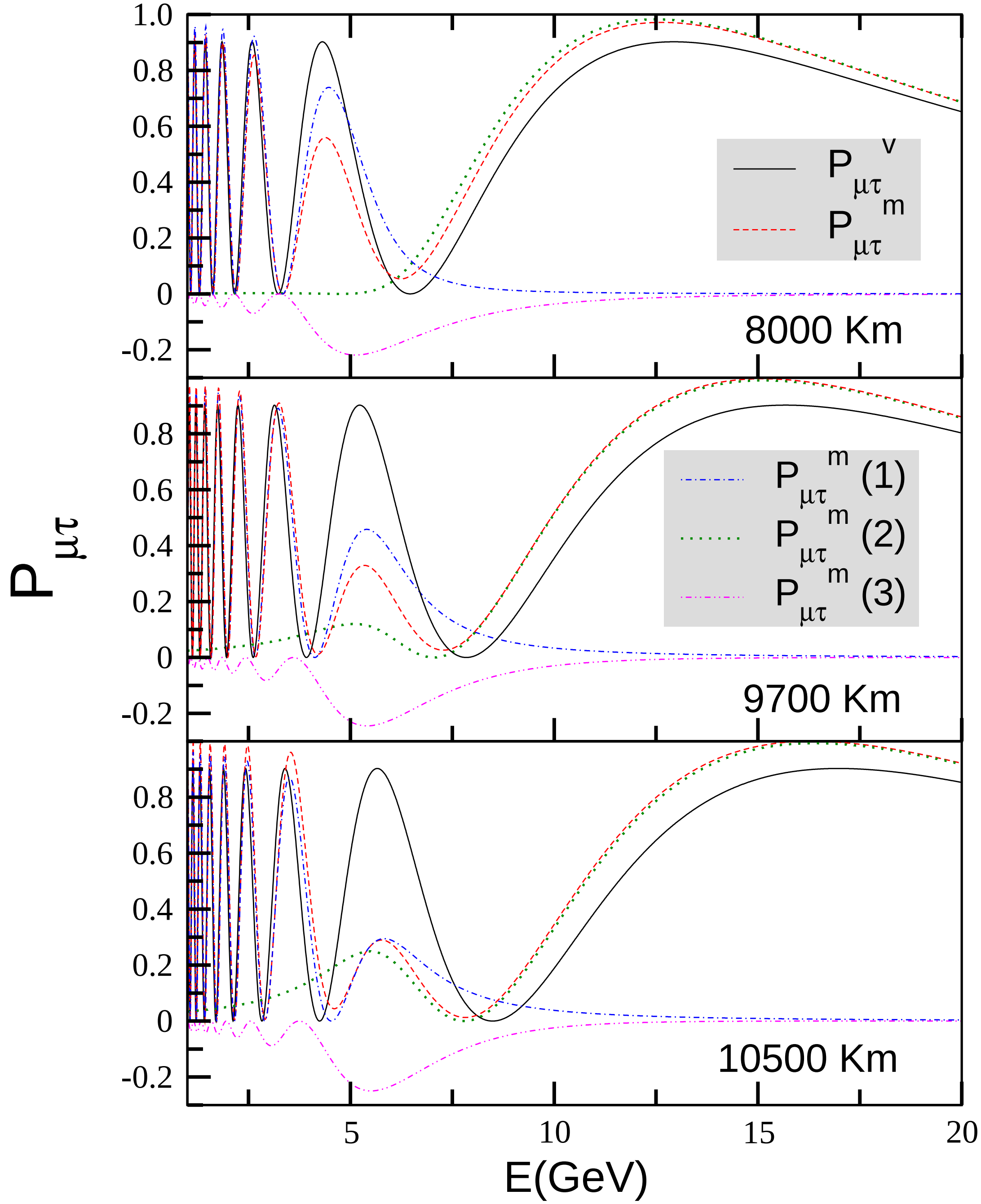
But the breaking of this degeneracy has an important impact on $\nu_\mu \to \nu_\tau$ oscillations. Eventhough the elastic scattering cross sections of ν_μ and ν_τ with ordinary matter are identical, it is still possible to get large matter effects in $\nu_\mu \to \nu_\tau$ oscillations because the matter term breaks the degeneracy present in vacuum oscillations.

Once again, it is reasonable to suppose that matter effects are the largest in the neighbourhood of resonance. But for what values of L will we get maximum matter effects? For that we look at each of the terms in the expression for $P_{\mu\tau}$.

For energies much below resonance energy, $A \ll \Delta_{31}$ and $\Delta_{31}^m \simeq \Delta_{31}$. Thus the first term is essentially

$P_{\mu\tau}^{vac}$ and the second and third terms are negligible. As we approach the resonance, θ_{13}^m approaches $\pi/4$. This leads to $\cos^2\theta_{13}^m$ decreasing sharply and $\sin^2\theta_{13}^m$ increasing sharply.

However, if we are in the neighbourhood of a vacuum peak of $P_{\mu\tau}$, then the effect of the decrease of the first term is stronger than the effect of the increase of the second term. This is illustrated in the following figure.



$P_{\mu\tau}$
E(GeV)
$P_{\mu\tau}^{v}$
$P_{\mu\tau}^{m}$
8000 Km
$P_{\mu\tau}^{m}$ (1)
$P_{\mu\tau}^{m}$ (2)
$P_{\mu\tau}^{m}$ (3)
9700 Km
10500 Km

Thus we are led to the condition $E_{res} = E_{peak}^{vac}$ for maximum matter effect in $\nu_\mu \rightarrow \nu_\tau$ oscillations. This in turn leads to the condition

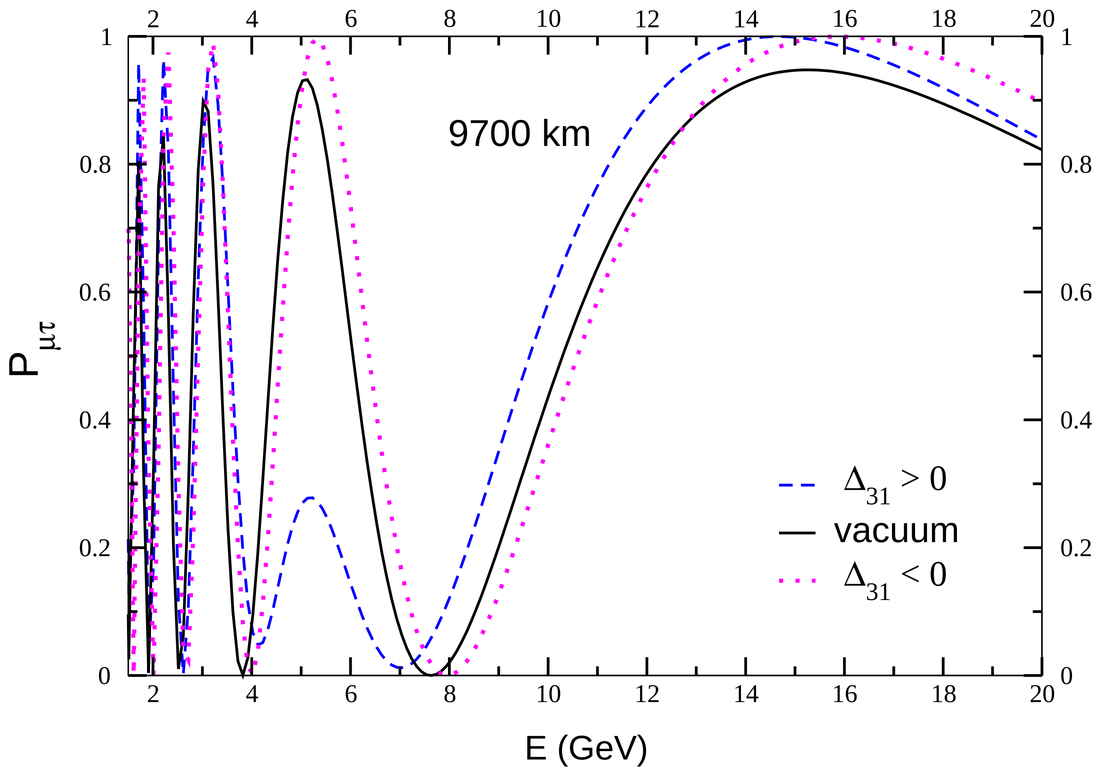
$$\rho L_{\mu\tau}^{max} \simeq (2\mathrm{p}+1)\,\pi\,5.18 \times 10^3\,(\cos 2\theta_{13})\ \mathrm{Km\ gm/cc.}$$

Because $\cos 2\theta_{13}$ has very weak dependence on θ_{13} for small values, it *may be* possible to observe this resonant amplification of matter effects even if θ_{13} is quite small. This, of course, requires a specialized τ detectors and intense neutrino sources because the τ production cross section is very small.

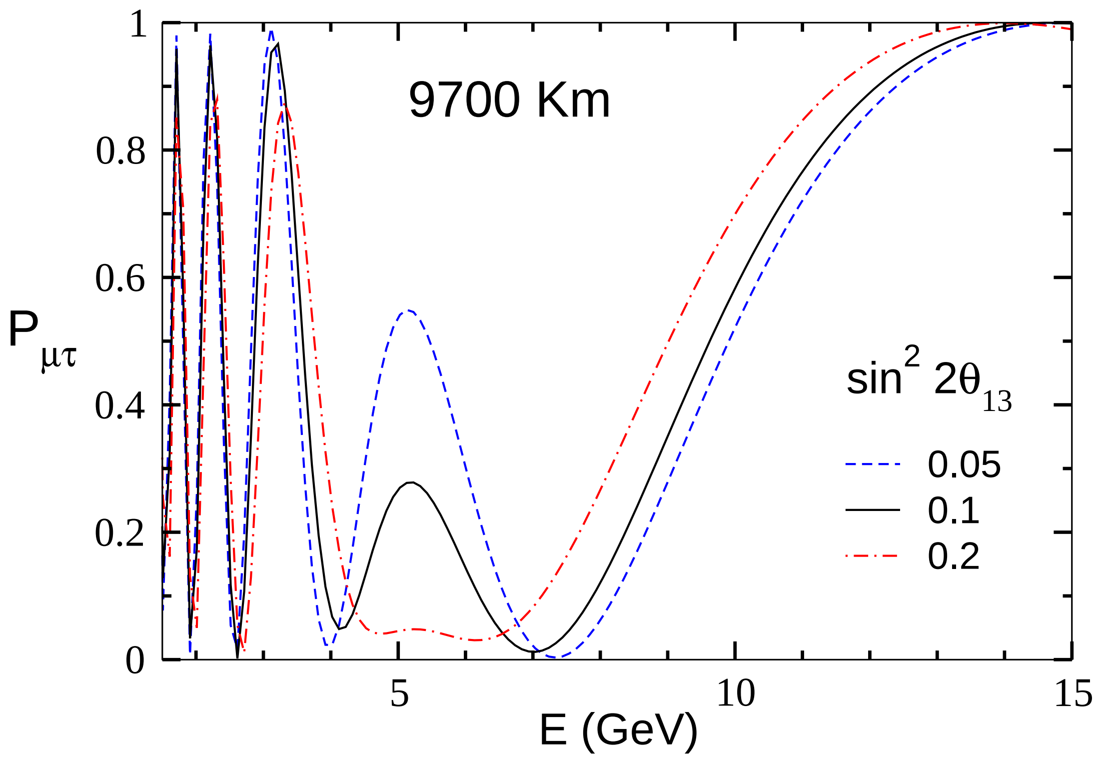
Incorporating the condition $E_{res} = E_{peak}^{vac}$ we get

$$\begin{aligned} \Delta \mathrm{P}_{\mu\tau} &= \mathrm{P}_{\mu\tau}^{\mathrm{m}} - \mathrm{P}_{\mu\tau}^{\mathrm{v}} \\ &= \cos^4 \left[\sin 2\theta_{13}(2p+1)\frac{\pi}{4} \right] - 1 \end{aligned}$$

For $p = 1$, $E_{res} = E_{peak}^{vac}$ osccurs for $L = 9700$ Km, for $\theta_{13} = 10^\circ$. This is illustrated in the first figure. Near $E = E_{res} = 5$ GeV, $P_{\mu\tau}$ falls from 1 to 0.3.



We can see the θ_{13} dependence of this resonance amplified matter effect in



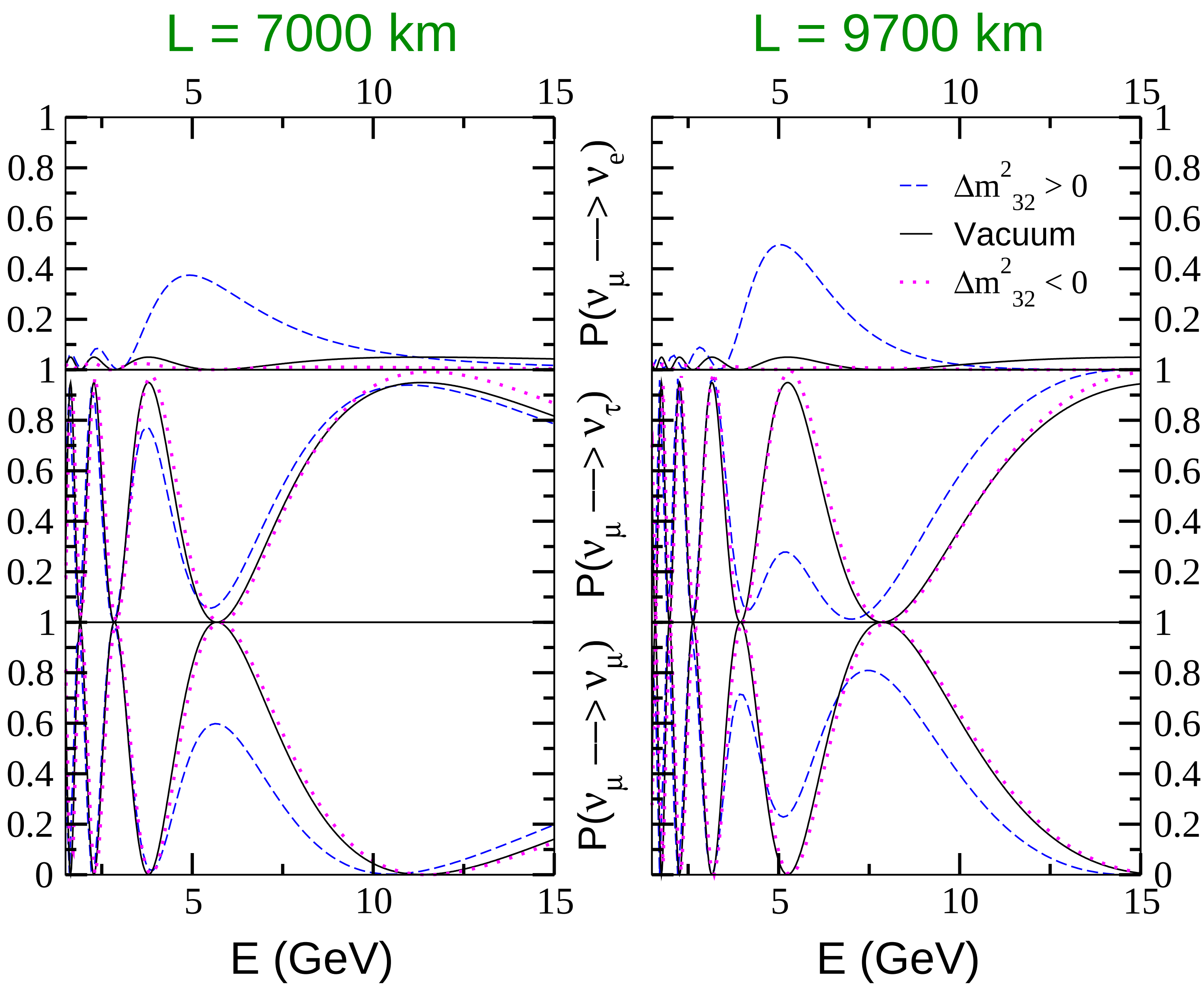
Even for $\sin^2 2\theta_{13} = 0.05$, the effect is quite dramatic. Observing this effect will be quite difficult because the τ production cross sections are quite small for $E_\nu \simeq 5$ GeV.

For $p = 0$, this matching condition $E_{res} = E^{vac}_{peak}$ occurs for $L = 4400$ Km, but $\Delta P_{\mu\tau}$ is *one-tenth* compared to $p = 1$ case.

Matter Effects in $P_{\mu\mu}$

Matter effects in $\nu_\mu \to \nu_e$ and $\nu_\mu \to \nu_\tau$ oscillations together lead to matter effects in $P_{\mu\mu}$.

$$\Delta P_{\mu\mu} = -\Delta P_{\mu e} - \Delta P_{\mu\tau}$$


L = 7000 km
L = 9700 km
$\Delta m^2_{32} > 0$
Vacuum
$\Delta m^2_{32} < 0$
$P(\nu_\mu \text{--->} \nu_e)$
$P(\nu_\mu \text{--->} \nu_\tau)$
$P(\nu_\mu \text{--->} \nu_\mu)$
E (GeV)
E (GeV)

From the previous figure, we notice the following points.

1. Increase in $P^m_{\mu e}$ occurs over a broad energy range for large path lengths
2. Decrease in $P^m_{\mu\tau}$ occurs over a rather narrow range for the same pathlengths.

Thus if we choose a condition $E_{res} = E^{vac}_{peak}$, then we have large positive $\Delta P_{\mu e}$ and large negative $\Delta P_{\mu\tau}$. These two effects nearly cancel in $\Delta P_{\mu\mu}$.

There is no energy region where both $\Delta P_{\mu e}$ and $\Delta P_{\mu\tau}$ are large and have the same sign.

So we look for an energy region where they both have the same sign but one is large.

Given that $P^{m}_{\mu e}$ has substantial increase over a large energy range, it is easier to find conditions under which $\Delta P_{\mu e}$ is large and $\Delta P_{\mu\tau}$ is small and both are positive.

This occurs when E_{res} coincides with the energy where $P^{vac}_{\mu\mu}$ has a peak.

The condition for pathlength where it occurs is given by

$$\rho L_{\mu\mu}^{max} = p\pi \times 10^4 (\cos 2\theta_{13}) \; Km \; gm/cc.$$

Unlike in the case of $\nu_\mu \to \nu_e$, here the pathlength condition for resonace amplification of matter effects has a very weak dependence on θ_{13}. Hence $\nu_\mu \to \nu_\mu$ survival probability is suitable to study these effects even for small values of θ_{13}.

For $p = 1$, this pathlength is 7000 Km. For this pathlength, $\Delta P_{\mu e} = 0.3$ and $\Delta P_{\mu\tau} = 0.1$ leading to $\Delta P_{\mu\mu} = -0.4$.

Luckily this energy region falls in exactly the range where INO has good muon detection capability.

We computed the number of muon events for an exposure time of 1000 Kton-Year. For the energy range 5-10 GeV and pathlength range 6000 to 10000 Km, we expect **261** events in the case of vacuum oscillations and **204** events in the case of matter oscillations.