Probing the Neutrino Mass Hierarchy in Future Detectors using the Atmospheric ν_{μ} Survival Rate

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 Importance of the Mass Hierarchy and future prospects of its detection



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- Matter effects in $P_{\mu e}, P_{\mu \tau}$ and $P_{\mu \mu}$



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Introduction . . .

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- About a decade ago, the goals were understanding the origin of the solar and atmospheric anomalies and deficits, and whether these were due to oscillations or were related to our incomplete understanding of the sources.

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- The present goal is to determine, as precisely as we can, the elements of the leptonic mixing matrix and the masses/mass-squared differences of neutrinos.
- Precision allows us to identify or exclude special valued mixing angles like $\theta_{13} = 0^{\circ}, \theta_{23} = 45^{\circ}$, and special relations between the quark and lepton sectors like $\theta_{12} + \theta_C = 45^{\circ}$, check for unitarity of 3 generations, non-standard interactions, decoherence scenarios.....

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- A multi-pronged effort to acheive these goals is underway via various operating, planned and proposed experiments, *e.g.* long-baseline, reactor, atmospheric, solar, beta-decay, *v*-less double beta decay, large-mass water cerenkov detectors

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• The 3×3 neutrino mixing matrix U in the MNS parametrization is:

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ ($\theta_{ij} = \text{mixing angle}$ between i^{th} & j^{th} mass states). $\delta = \text{CP}$ phase. Summary of present knowledge :



ν The Mass Hierarchy, its Significance and Detection

So far we only know $|\Delta m_{31}^2|$ and not its Sign



The Mass Hierarchy ...

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- A large class of GUTS use the Type I seesaw mechanism to unify quarks and leptons. Several positive features are lost if in such models the neutrino hierarchy is *inverted* rather than *normal* C. Albright, Phys. Lett. B599, 285 (2004)

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- It would also favour theories utilising the Type II seesaw mechanism with additional Higgs triplets.
- The type of hierarchy impacts the effectiveness of leptogenesis in most theoretical models.

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 - $\nu_{\mu} \rightarrow \nu_{e}$ > HK, UNO Megaton Water Cerenkov, via atmospheric $\nu_{\mu} \rightarrow \nu_{e}$
 - Nu Factories substantial improvement in bounds

The Mass Hierarchy . . .

• Detection of the hierarchy via the $\nu_{\mu} \rightarrow \nu_{e}$ channel, at not-so-long baselines, is hampered by a $\delta_{CP} - sign(\Delta_{31})$ degeneracy which reduces sensitivity. Additionally, in "off-resonance" situations, there is a $\theta_{13} - sign(\Delta_{31})$ that must be taken into account.

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- Overcoming this in superbeam experiments would require using either multiple oscillation channels, 2 baselines, different energies, the magic baseline, 2 off-axis locations Or a combination of these stategies.

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These degeneracies are absent in methods which utilise the muon survival probability. Probing the Neutrino Mass Hierarchy.



• Vacuum 1-MSD limit ($\Delta m^2_{21} = 0$):

$$\mathcal{P}_{\mu e} = \sin^2 \theta_{23} \sin^2(2\theta_{13}) \sin^2 \left[\Delta_{31} L/E\right]$$



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In matter,

 $\mathcal{P}^{m}_{\nu_{\mu} \to \nu_{e}} = \sin^{2} \theta_{23} \sin^{2} (2\theta^{m}_{13}) \sin^{2} \left[\Delta^{m}_{31} L/E \right]$



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• and
$$\Delta_{31}^m = \Delta_{31} \sqrt{(\frac{A}{\delta m_{31}^2} - \cos 2\theta_{13})^2 + \sin^2 2\theta_{13}}$$

Probing the Neutrino Mass Hierarchy . . .



• We note that: $\mathcal{P}_{\mu e}^{m}$ is maximized not just at resonance but when the combination $\sin^{2}(2\theta_{13}^{m})\sin^{2}[\Delta_{31}^{m}L/E]$ is maximal, *i.e*i.e. when $E = E_{res} = E_{peak}^{m}$.



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- For Earth baselines, this translates to:

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• The *L* where these (p = 0) maxima occur are 7600 km for $sin^2 2\theta_{13} = 0.2$, 10200 km for $sin^2 2\theta_{13} = 0.1$ and 11200 km for $sin^2 2\theta_{13} = 0.05$.



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Matter effects in $P_{\mu\tau}$ and $P_{\mu\mu}$

Solution Vacuum 1-MSD limit ($\Delta m_{21}^2 = 0$) :

$$\mathcal{P}_{\mu e} = \sin^2 \theta_{23} \sin^2 (2\theta_{13}) \sin^2 \left[1.27 \Delta m_{31}^2 L/E \right]$$

$$\mathcal{P}_{\mu\tau} = \cos^4 \theta_{13} \sin^2(2\theta_{23}) \sin^2\left[1.27\Delta m_{31}^2 L/E\right]$$

$$\mathcal{P}_{\mu\mu} = 1 - \mathcal{P}_{\mu e} - \mathcal{P}_{\mu au}$$

Matter effect on $P(\nu_{\mu} \rightarrow \nu_{\tau})$

The 1-MSD vacuum expression is:

$$\mathcal{P}^{v}_{\nu_{\mu} \to \nu_{\tau}} = \cos^2 \theta_{13} \sin^2 2\theta_{23} \sin^2(\Delta_{31})$$

$$-\cos^2\theta_{23}P^{vac}_{\nu_\mu\to\nu_e}$$

The 1-MSD analytic expression in matter is:

$$\mathcal{P}^{m}_{\nu_{\mu} \to \nu_{\tau}} = \cos^{2} \theta^{m}_{13} \sin^{2} 2\theta_{23} \sin^{2} [(\Delta_{31} + A + \Delta_{31}^{m})/2] + \sin^{2} \theta^{m}_{13} \sin^{2} 2\theta_{23} \sin^{2} [(\Delta_{31} + A - \Delta_{31}^{m})/2] - \cos^{2} \theta_{23} P^{mat}_{\nu_{\mu} \to \nu_{e}}$$



Maximizing the matter effect in $\mathcal{P}_{\mu\tau}$

•
$$E = E_{res} = E_{peak}^v$$
 leads to:

 $\Delta \mathcal{P}_{\mu\tau} \simeq \cos^4 [\sin 2\theta_{13}(2p+1)\pi/4] - 1$

(with $\sin^2 2\theta_{23} = 1$, $\cos^2 \theta_{13}$, $\cos 2\theta_{13} \simeq 1$).

The maximum matter effect condition for L is:

 $[\rho L]_{\mu\tau}^{max} = (2p+1)\pi \times 5.18 \times 10^3 \times \cos 2\theta_{13} \ km \ gm/cc$

The maximum matter effect occurs for L = 9700 km for p=1 & $\sin^2 2\theta_{13} = 0.1$ (9300 km, 9900 km for 0.2, 0.05).

Summary of Conditions for Large Matter effects ...

Thus, conditions for maximizing the matter effects at very long baselines are:

• $[\rho L]_{\mu e}^{\max} = \frac{(2p+1)\pi 5.18 \times 10^3}{\tan 2\theta_{13}}$ Km gm/cc. • $[\rho L]_{\mu\tau}^{\max} = (2p+1)\pi 5.18 \times 10^3 \cos 2\theta_{13}$ Km gm/cc. • $[\rho L]_{\mu\mu}^{\max} = p\pi \times 10^4 \cos 2\theta_{13}$ Km gm/cc.

Plot of Probabilities . . .

R. Gandhi et al., Phys. Rev. Lett. 94, 051801 (2005); hep-ph/0411252



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Survival Probability : θ_{13} sensitivity . . .



Results for an Iron Calorimeter type detector

Results : Iron Calorimeter, 1000 kt-yr . . .

L = 6000 to 9700 Km, E = 5 to 10 GeV



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L = 8000 to 10700 Km, E = 4 to 8 GeV



July 07, 2005 BNL

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Fe Calorimeter, 1000kt-yr, $\Delta \chi^2$ and σ sensitivity . . .

Choice of the optimal ranges for μ^- events for 1000 kiloton-yr: Range 1 : E = 5 - 10 GeV and L = 6000 - 9700 Km (4 selected bins)

$\sin^2 2\theta_{13}$	$N_{\rm vac}$	$\mathrm{N}_{\mathrm{mat}}(\Delta m^2_{31}>0)$	$\Delta\chi^2$	sensitivity
0.05	260	227	5.35	1.14 σ
0.1	261	204	19.76	3.44 σ
0.2	263	163	86.40	8.6σ

Range 2 : E = 4 - 8 GeV and L = 8000 - 10700 Km $(Log_{10}(L/E)=3.21-3.44)$

$\sin^2 2 heta_{13}$	$N_{\rm vac}$	$N_{\rm mat}(\Delta m_{31}^2 > 0)$	$\Delta\chi^2$	sensitivity
0.05	23.3	42.6	8.7	2.94σ
0.1	23.3	62.7	24.79	4.97σ
0.2	24.5	104.5	61.24	7.82σ

Results for a Megaton water cerenkov detector

Megaton Water Cerenkov, 1.8 Mt-yr exposure ...

L = 6000 to 9700 km, E = 5 to 10 GeV



Megaton Water Cerenkov, 1.8 Mt-yr exposure ...

 $Log_{10} L/E = 3.215 - 3.322 \text{ km/GeV}$



In Preparation

Megaton Water Cerenkov, 1.8 Mt-yr, $\Delta \chi^2$ and σ sensitivity ...

Choice of the optimal ranges for $\mu^- + \mu^+$ events for 1.8 megaton-yr: Range 1 : E = 5 - 10 GeV and L = 6000 - 9700 Km

$\sin^2 2 heta_{13}$	$N_{\rm vac}$	$\mathrm{N}_{\mathrm{mat}}(\Delta m_{31}^2 > 0)$	$\Delta\chi^2$	sensitivity
0.05	731.8	664.4	6.86	2.62 σ
0.1	735.3	616.0	23.09	4.8 σ
0.2	743.0	530.1	85.50	9.25σ

Range 2 : E = 4 - 8 GeV and L = 8000 - 10700 Km $Log_{10}(L/E) = 3.215 - 3.322 \text{Km/GeV}$

$\sin^2 2 heta_{13}$	$N_{\rm vac}$	$N_{\rm mat}(\Delta m_{31}^2 > 0)$	$\Delta\chi^2$	sensitivity
0.05	40.9	73.5	14.38	3.79σ
0.1	42.9	107.2	38.45	6.2 σ
0.2	48.7	178.3	94.22	9.71 <i>σ</i>



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- The mass hierarchy is a powerful discriminator between various classes of unification theories and its determination is a key goal for the future.
- Future superbeam experiments using the $\nu_{\mu} \rightarrow \nu_{e}$ channel for hierarchy determination will find it difficult to acheive the required sensitivities due to degeneracies
- It is worthwhile to explore the $\nu_{\mu} \rightarrow \nu_{\mu}$ channel for this purpose since it is largely free of degeneracies.



- Large matter effects are not only confined to $P_{\mu e}$ but also arise in $P_{\mu \tau}$ at GeV energies at very long Earth baselines. Both must be properly considered when evaluating the event-rates for experiments measuring muon survival.
- The effects discussed above are significantly sensitive to θ_{13} and Sign of Δm_{31}^2 , determination of which are outstanding problems of neutrino physics.
- We have tried to show that there is a good possibility that one can determine the Sign of Δm_{31}^2 using atmospheric neutrinos in a large mass iron calorimeter with charge id as well as using a Megaton water Cerenkov detector

Matter effect in $\nu_{\mu} \rightarrow \nu_{\tau}$

 \blacklozenge Click here for $P_{\mu\tau} \rightsquigarrow$

♦ The animation shows the matter effects building up in $P_{\mu\tau}$ for L = 6000 to 10500 km. ♦ Maximum matter effects seen at 9700 km.

 \diamond The term wise break up of probability is also depicted in the animation.

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To Summary page

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