Probing the Neutrino Mass Hierarchy in Future Detectors using the Atmospheric $\nu_\mu$ Survival Rate

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Introduction
Outline

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- Importance of the Mass Hierarchy and future prospects of its detection
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Matter effects in $P_{\mu e}$, $P_{\mu \tau}$ and $P_{\mu \mu}$
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- Results: Detecting the Mass Hierarchy in atmospheric $\nu_\mu$ events
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Conclusions
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About a decade ago, the goals were understanding the origin of the solar and atmospheric anomalies and deficits, and whether these were due to oscillations or were related to our incomplete understanding of the sources.
The present goal is to determine, as precisely as we can, the elements of the leptonic mixing matrix and the masses/mass-squared differences of neutrinos.
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Precision allows us to identify or exclude special valued mixing angles like $\theta_{13} = 0^o$, $\theta_{23} = 45^o$, and special relations between the quark and lepton sectors like $\theta_{12} + \theta_C = 45^o$, check for unitarity of 3 generations, non-standard interactions, decoherence scenarios......
Note that measurements in the lepton sector are unhindered by hadronic uncertainties which are inherent in the quark sector. This enables clean tests of flavour physics limited only by experimental precision.
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A multi-pronged effort to achieve these goals is underway via various operating, planned and proposed experiments, e.g. long-baseline, reactor, atmospheric, solar, beta-decay, $\nu$-less double beta decay, large-mass water Cerenkov detectors.
\[ (\nu_\alpha) = U(\nu_i) \text{ with } i = 1, 2, 3 \text{ and } \alpha = e, \mu, \tau. \]
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\]

The \(3 \times 3\) neutrino mixing matrix \(U\) in the MNS parametrization is:

\[
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}s_{13}
\end{pmatrix}
\]

where \(c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij}\) (\(\theta_{ij}\) = mixing angle between \(i^{th}\) & \(j^{th}\) mass states). \(\delta = \text{CP phase.}\)
Summary of present knowledge:
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- \( \tan^2 \theta_{12} = 0.39_{-0.04}^{+0.05} \) \( \text{solar, reactor} \)
- \( \tan^2 \theta_{23} = 1.0_{-0.3}^{+0.3} \) \( \text{atmospheric, K2K} \)
- \( \theta_{13} < 12^\circ @3\sigma \) \( \text{reactor, atmospheric} \)
- \( \delta m^2_{21} = 8.2_{-0.3}^{+0.3} \times 10^{-5} \text{eV}^2 \) \( \text{solar, reactor} \)
- \( |\delta m^2_{31}| = 2.2_{-0.4}^{+0.6} \times 10^{-3} \text{eV}^2 \) \( \text{atmospheric, K2K} \)
- \( m_\beta < 2.2 \text{ eV} \) \( \text{beta decay} \)
- \( m_{\beta\beta0\nu} < 0.3 \text{ eV} \) \( \nu \) less double beta decay
- \( \sum m_i < 1.6 - 0.7 \text{ eV} \) \( \text{Precision Cosmology} \)
- \( \delta_{CP} \) is unknown
So far we only know $|\Delta m_{31}^2|$ and not its Sign
The importance of the mass hierarchy lies in its capability to discriminate between various types of models for unification, and thus in its ability to narrow the focus of the quest for physics beyond the Standard Model.
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A large class of GUTS use the Type I seesaw mechanism to unify quarks and leptons. Several positive features are lost if in such models the neutrino hierarchy is inverted rather than normal.

An *inverted* hierarchy on the other hand would generally require $m_1, m_2$ to be *degenerate*, hinting towards an *additonal global symmetry in the lepton sector*. 
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It would also favour theories utilising the *Type II seesaw mechanism* with additional *Higgs triplets*. 
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The type of hierarchy impacts the effectiveness of leptogenesis in most theoretical models.
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- T2K superbeam, 295 km, $<E_\nu> = 0.76$ GeV, via $\nu_\mu \rightarrow \nu_e$
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➢ **Nu Factories** substantial improvement in bounds
Detection of the hierarchy via the $\nu_\mu \rightarrow \nu_e$ channel, at not-so-long baselines, is hampered by a

$$\delta_{CP} = \text{sign}(\Delta_{31})$$

degeneracy which reduces sensitivity. Additionally, in “off-resonance” situations, there is a

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Overcoming this in superbeam experiments would require using either multiple oscillation channels, 2 baselines, different energies, the magic baseline, 2 off-axis locations or a combination of these strategies.

Minakata et al, Beavis et al, Donini et al, Barger et al, Huber et al, Burguet-Castell et al, O. Mena Requejo et al.....
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These degeneracies are absent in methods which utilise the muon survival probability.
Matter effects in $P_{\mu e}$

**Vacuum 1-MSD limit ($\Delta m^2_{21} = 0$):**

$$P_{\mu e} = \sin^2 \theta_{23} \sin^2 (2\theta_{13}) \sin^2 [\Delta_{31} L/E]$$
Matter effects in $P_{\mu e}$

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$$P_{\nu_{\mu} \rightarrow \nu_e} = \sin^2 \theta_{23} \sin^2(2\theta_{13}^m) \sin^2 [\Delta_{31}^m L/E]$$
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where

$$\sin^2 2\theta_{13}^m = \frac{\sin^2 2\theta_{13}}{(\frac{A}{\delta m_{31}^2} - \cos 2\theta_{13})^2 + \sin^2 2\theta_{13}}$$
Matter effects in $P_{\mu e}$

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**where**

$$\sin^2 2\theta_{13}^m = \frac{\sin^2 2\theta_{13}}{(\frac{A}{\delta m_{31}^2} - \cos 2\theta_{13})^2 + \sin^2 2\theta_{13}}$$

**and**

$$\Delta_{31}^m = \Delta_{31} \sqrt{(\frac{A}{\delta m_{31}^2} - \cos 2\theta_{13})^2 + \sin^2 2\theta_{13}}$$
We note that: $P_{\mu e}^m$ is maximized not just at resonance but when the combination $\sin^2(2\theta_{13}^m) \sin^2 [\Delta_{31}^m L/E]$ is maximal, i.e. when $E = E_{res} = E_{peak}^m$. 
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For Earth baselines, this translates to:

$$ [\rho L]_{\mu e}^{max} = \frac{(2p + 1)\pi \times 5.18 \times 10^3}{\tan 2\theta_{13}} \quad \text{km gm/cc} $$

**Note sensitivity to $\theta_{13}$.**
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The $L$ where these ($p = 0$) maxima occur are 7600 km for $\sin^2 2\theta_{13} = 0.2$, 10200 km for $\sin^2 2\theta_{13} = 0.1$ and 11200 km for $\sin^2 2\theta_{13} = 0.05$. 
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Co-relations and degeneracies play an important role and reduce sensitivity when $\mathcal{P}_{\mu e}^m$ is used to determine the hierarchy.
Matter effects in $P_{\mu\tau}$ and $P_{\mu\mu}$

Vacuum 1-MSD limit ($\Delta m_{21}^2 = 0$):

\[P_{\mu e} = \sin^2 \theta_{23} \sin^2 (2\theta_{13}) \sin^2 [1.27\Delta m_{31}^2 L/E]\]

\[P_{\mu \tau} = \cos^4 \theta_{13} \sin^2 (2\theta_{23}) \sin^2 [1.27\Delta m_{31}^2 L/E]\]

\[P_{\mu \mu} = 1 - P_{\mu e} - P_{\mu \tau}\]
Matter effect on $P(\nu_\mu \to \nu_\tau)$

- The 1-MSD vacuum expression is:

$$P_{\nu_\mu \to \nu_\tau}^v = \cos^2 \theta_{13} \sin^2 2\theta_{23} \sin^2(\Delta_{31})$$

- The 1-MSD analytic expression in matter is:

$$P_{\nu_\mu \to \nu_\tau}^m = \cos^2 \theta_{13}^m \sin^2 2\theta_{23} \sin^2[\Delta_{31} + A + \Delta_{31}^m]/2$$

$$+ \sin^2 \theta_{13}^m \sin^2 2\theta_{23} \sin^2[\Delta_{31} + A - \Delta_{31}^m]/2$$

$$- \cos^2 \theta_{23} P_{\nu_\mu \to \nu_e}^\text{mat}$$
Maximizing the matter effect in $\mathcal{P}_{\mu\tau}$

- $E = E_{res} = E_{peak}^v$ leads to:

$$\Delta\mathcal{P}_{\mu\tau} \simeq \cos^4[\sin 2\theta_{13}(2p + 1)\pi/4] - 1$$

(with $\sin^2 2\theta_{23} = 1$, $\cos^2 \theta_{13}$, $\cos 2\theta_{13} \simeq 1$).

- The maximum matter effect condition for $L$ is:

$$[\rho L]_{\mu\tau}^{max} = (2p + 1)\pi \times 5.18 \times 10^3 \times \cos 2\theta_{13} \text{ km gm/cc}$$

The maximum matter effect occurs for $L = 9700$ km for $p=1$ & $\sin^2 2\theta_{13} = 0.1$ (9300 km, 9900 km for 0.2, 0.05).
Thus, conditions for maximizing the matter effects at very long baselines are:

\[
\begin{align*}
[ho L]_{\mu e}^{\text{max}} &= \frac{(2p+1)\pi 5.18 \times 10^3}{\tan 2\theta_{13}} \quad \text{Km gm/cc.} \\
[ho L]_{\mu\tau}^{\text{max}} &= (2p + 1)\pi 5.18 \times 10^3 \cos 2\theta_{13} \quad \text{Km gm/cc.} \\
[ho L]_{\mu\mu}^{\text{max}} &= p\pi \times 10^4 \cos 2\theta_{13} \quad \text{Km gm/cc.}
\end{align*}
\]
Plot of Probabilities . . .

Plot of Probabilities...
Survival Probability: $\theta_{13}$ sensitivity

R. Gandhi et al., hep-ph/0411252
Results for an Iron Calorimeter type detector
Results: Iron Calorimeter, 1000 kt-yr...

L = 6000 to 9700 Km, E = 5 to 10 GeV

$\sin^2 2\theta_{13} = 0.1$

$\Delta_{31} = 0.002$ eV$^2$

$N_{\mu}^m = 204$

$N_{\mu}^\nu = 261$

$N_{\mu}^{m} = 103$

$N_{\mu}^{\nu} = 105$

R. Gandhi et al., hep-ph/0411252
\[ \Delta_{31} = 0.002 \text{ eV}^2 \]

R. Gandhi et al., hep-ph/0411252
Results: Iron Calorimeter, 1000 kt-yr ...
Choice of the optimal ranges for $\mu^-$ events for 1000 kiloton-yr:

Range 1 : $E = 5 - 10$ GeV and $L = 6000 - 9700$ Km (4 selected bins)

<table>
<thead>
<tr>
<th>$\sin^2 2\theta_{13}$</th>
<th>$N_{\text{vac}}$</th>
<th>$N_{\text{mat}} (\Delta m^2_{31} &gt; 0)$</th>
<th>$\Delta \chi^2$</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>260</td>
<td>227</td>
<td>5.35</td>
<td>1.14$\sigma$</td>
</tr>
<tr>
<td>0.1</td>
<td>261</td>
<td>204</td>
<td>19.76</td>
<td>3.44$\sigma$</td>
</tr>
<tr>
<td>0.2</td>
<td>263</td>
<td>163</td>
<td>86.40</td>
<td>8.6$\sigma$</td>
</tr>
</tbody>
</table>

Range 2 : $E = 4 - 8$ GeV and $L = 8000 - 10700$ Km

$(\log_{10}(L/E)) = 3.21 - 3.44$)

<table>
<thead>
<tr>
<th>$\sin^2 2\theta_{13}$</th>
<th>$N_{\text{vac}}$</th>
<th>$N_{\text{mat}} (\Delta m^2_{31} &gt; 0)$</th>
<th>$\Delta \chi^2$</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>23.3</td>
<td>42.6</td>
<td>8.7</td>
<td>2.94$\sigma$</td>
</tr>
<tr>
<td>0.1</td>
<td>23.3</td>
<td>62.7</td>
<td>24.79</td>
<td>4.97$\sigma$</td>
</tr>
<tr>
<td>0.2</td>
<td>24.5</td>
<td>104.5</td>
<td>61.24</td>
<td>7.82$\sigma$</td>
</tr>
</tbody>
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Results for a Megaton water cerenkov detector
L = 6000 to 9700 km, E = 5 to 10 GeV

$L = 6000 \text{ to } 9700 \text{ km, } E = 5 \text{ to } 10 \text{ GeV}$

In Preparation
Megaton Water Cerenkov, 1.8 Mt-yr exposure...

Log\textsubscript{10} L/E = 3.215 - 3.322 km/GeV

In Preparation
Choice of the optimal ranges for $\mu^- + \mu^+$ events for 1.8 megaton-yr:

Range 1: $E = 5 - 10$ GeV and $L = 6000 - 9700$ Km

<table>
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<tr>
<th>$\sin^2 2\theta_{13}$</th>
<th>$N_{\text{vac}}$</th>
<th>$N_{\text{mat}}(\Delta m^2_{31} &gt; 0)$</th>
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<th>sensitivity</th>
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</thead>
<tbody>
<tr>
<td>0.05</td>
<td>731.8</td>
<td>664.4</td>
<td>6.86</td>
<td>$2.62\sigma$</td>
</tr>
<tr>
<td>0.1</td>
<td>735.3</td>
<td>616.0</td>
<td>23.09</td>
<td>$4.8 \sigma$</td>
</tr>
<tr>
<td>0.2</td>
<td>743.0</td>
<td>530.1</td>
<td>85.50</td>
<td>$9.25\sigma$</td>
</tr>
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</table>

Range 2: $E = 4 - 8$ GeV and $L = 8000 - 10700$ Km

$\log_{10}(L/E) = 3.215 - 3.322$ Km/GeV

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<tbody>
<tr>
<td>0.05</td>
<td>40.9</td>
<td>73.5</td>
<td>14.38</td>
<td>$3.79\sigma$</td>
</tr>
<tr>
<td>0.1</td>
<td>42.9</td>
<td>107.2</td>
<td>38.45</td>
<td>$6.2 \sigma$</td>
</tr>
<tr>
<td>0.2</td>
<td>48.7</td>
<td>178.3</td>
<td>94.22</td>
<td>$9.71\sigma$</td>
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*Precision measurements of neutrino mass differences and mixing angles* are the dominant focus of experimental activity in neutrino physics for the next 15-20 years.
**To Sum Up...**

- **Precision measurements of neutrino mass differences and mixing angles** are the dominant focus of experimental activity in neutrino physics for the next 15-20 years.

- The **mass hierarchy** is a powerful discriminator between various classes of unification theories and its determination is a key goal for the future.
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- Future superbeam experiments using the $\nu_\mu \rightarrow \nu_e$ channel for hierarchy determination will find it difficult to achieve the required sensitivities due to degeneracies.
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- Precision measurements of neutrino mass differences and mixing angles are the dominant focus of experimental activity in neutrino physics for the next 15-20 years.

- The mass hierarchy is a powerful discriminator between various classes of unification theories and its determination is a key goal for the future.

- Future superbeam experiments using the $\nu_\mu \rightarrow \nu_e$ channel for hierarchy determination will find it difficult to achieve the required sensitivities due to degeneracies.

- It is worthwhile to explore the $\nu_\mu \rightarrow \nu_\mu$ channel for this purpose since it is largely free of degeneracies.
To Sum Up . . .

- Large matter effects are not only confined to $P_{\mu e}$ but also arise in $P_{\mu \tau}$ at GeV energies at very long Earth baselines. Both must be properly considered when evaluating the event-rates for experiments measuring muon survival.

- The effects discussed above are significantly sensitive to $\theta_{13}$ and Sign of $\Delta m_{31}^2$, determination of which are outstanding problems of neutrino physics.

- We have tried to show that there is a good possibility that one can determine the Sign of $\Delta m_{31}^2$ using atmospheric neutrinos in a large mass iron calorimeter with charge id as well as using a Megaton water Cerenkov detector.
Matter effect in $\nu_\mu \rightarrow \nu_\tau$

- The animation shows the matter effects building up in $P_{\mu \tau}$ for $L = 6000$ to $10500$ km.
- Maximum matter effects seen at $9700$ km.
- The term wise break up of probability is also depicted in the animation.
Matter effect in $\nu_\mu \rightarrow \nu_\mu$

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