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NEUTRINO MASS MATRIX A LA MODE

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With the accumulation of many years of solar and atmospheric neutrino oscillation data, the approximate form of the 3×3 neutrino mixing matrix is now known. What is not known is the (presumably Majorana) neutrino mass matrix $\mathcal{M}_\nu$ itself. In this chapter, the approximate form of $\mathcal{M}_\nu$ is derived, leading to seven possible neutrino mass patterns: three have the normal hierarchy, two have the inverse hierarchy, and two have three nearly degenerate masses. The generalization of this to allow $U_{e3} \neq 0$ with maximal CP violation is also discussed. A specific automatic realization of this $\mathcal{M}_\nu$ from radiative corrections of an underlying non-Abelian discrete A_4 symmetry in the context of softly broken supersymmetry is presented.

KeyWords: Neutrino Mass Matrix; Normal Hierarchy; Inverted Hierarchy; Quasidegeneracy; Maximal CP Violation; A_4 Symmetry

With the recent addition of KamLAND (Kamioka Liquid Scintillator Anti-Neutrino Detector) data[1] together with the previous SNO (Sudbury Neutrino Observatory) neutral-current data[2], the overall picture of solar neutrino oscillations[3] is becoming very clear. Combined with the well-established atmospheric neutrino data[4], the 3×3 neutrino mixing matrix is now determined to a very good first approximation by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & -1/\sqrt{2} \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad \ldots(1)$$

where $\nu_{1,2,3}$ are assumed to be Majorana neutrino mass eigenstates. In the above, $\sin^2 2\theta_{atm} = 1$ is already assumed and θ is the solar mixing angle which is now known to be large but not maximal[5], i.e. $\tan^2\theta \simeq 0.4$. The U_{e3} entry has been assumed zero but it is only required experimentally to be small[6], i.e. $|U_{e3}| < 0.16$.

Denoting the masses of $\nu_{1,2,3}$ as $m_{1,2,3}$, the solar neutrino data[2,3] require that $m_2^2 > m_1^2$ with $\theta < \pi/4$, and in the case of the favored large-mixing-angle solution[5],

$$\Delta m^2_{sol} = m_2^2 - m_1^2 \simeq 5 \times 10^{-5} \text{ eV}^2. \quad \ldots(2)$$

The atmospheric neutrino data[4] require

$$|m_3^2 - m_{1,2}^2| \simeq 2.5 \times 10^{-3} \text{ eV}^2, \quad \ldots(3)$$

without deciding whether $m_3^2 > m_{1,2}^2$ or $m_3^2 < m_{1,2}^2$.

The big question now is what the neutrino mass matrix itself should look like. Of course it may be obtained by using eq.1, i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} c^2m_1 + s^2m_2 & sc(m_1-m_2)/\sqrt{2} & sc(m_1-m_2)/\sqrt{2} \\ sc(m_1-m_2)/\sqrt{2} & (s^2m_1+c^2m_2+m_3)/2 & (s^2m_1+c^2m_2-m_3)/2 \\ sc(m_1-m_2)/\sqrt{2} & (s^2m_1+c^2m_2-m_3)/2 & (s^2m_1+c^2m_2+m_3)/2 \end{pmatrix} \quad \ldots(4)$$

where $c \equiv \cos\theta$ and $s \equiv \sin\theta$. However this is not very illuminating theoretically. Instead it has been proposed[7] that it be rewritten in the form

$$\mathcal{M}_\nu = \begin{pmatrix} a+2b+2c & d & d \\ d & b & a+b \\ d & a+b & b \end{pmatrix}. \quad \ldots(5)$$

To satisfy $m_2^2 > m_1^2$ for $\theta < \pi/4$, there are 2 cases to be considered.

(I) For $a+2b+c > 0$ and $c < 0$,

$$m_1 = a+2b+c-\sqrt{c^2+2d^2}, \quad \ldots(6)$$
$$m_2 = a+2b+c+\sqrt{c^2+2d^2}, \quad \ldots(7)$$
$$m_3 = -a, \quad \ldots(8)$$
$$\tan\theta = \sqrt{2}d/(\sqrt{c^2+2d^2}-c). \quad \ldots(9)$$

(II) For $a+2b+c < 0$ and $c > 0$,

$$m_1 = a+2b+c+\sqrt{c^2+2d^2}, \quad \ldots(10)$$

$$m_2 = a+2b+c-\sqrt{c^2+2d^2}, \quad \ldots(11)$$
$$m_3 = -a, \quad \ldots(12)$$
$$\tan\theta = (\sqrt{c^2+2d^2}-c)/\sqrt{2}d. \quad \ldots(13)$$

Note that θ depends only on the ratio d/c, which must be of order unity. This shows the advantage of adopting the parametrization of eq.5. The constraints of eqs.2 and 3 are then realized by the following 7 different conditions on a, b, and c.

(1) $||a+2b+c|-\sqrt{c^2+2d^2}| \ll |a+2b+c| \ll |a|$, i.e. $|m_1| \ll |m_2| \ll |m_3|$.

(2) $\sqrt{c^2+2d^2} \ll |a+2b+c| \ll |a|$, i.e. $|m_1| \simeq |m_2| \ll |m_3|$.

(3) $|a+2b+c| \ll \sqrt{c^2+2d^2} \ll |a|$, i.e. $|m_1| \simeq |m_2| \ll |m_3|$.

(4) $|a|$, $\sqrt{c^2+2d^2} \ll |a+2b+c|$, i.e. $|m_3| \ll |m_1| \simeq |m_2|$.

(5) $|a|$, $|a+2b+c| \ll \sqrt{c^2+2d^2}$, i.e. $|m_3| \ll |m_1| \simeq |m_2|$.

(6) $\sqrt{c^2+2d^2} \ll ||a+2b+c|-|a|| \ll |a|$, i.e. $|m_1| \simeq |m_2| \simeq |m_3|$.

(7) $|a+2b+c| \ll \sqrt{c^2+2d^2} \simeq |a|$, i.e. $|m_1| \simeq |m_2| \simeq |m_3|$.

Cases (1) to (3) have the normal hierarchy. Cases (4) and (5) have the inverse hierarchy. Cases (6) and (7) have 3 nearly degenerate masses. The versatility of eq.5 has clearly been demonstrated.

The above 7 cases encompass all models of the neutrino mass matrix that have ever been proposed which also satisfy eq.1. They are also very useful for discussing the possibility of neutrinoless double beta ($\beta\beta_{0\nu}$) decay in the context of neutrino oscillations[8]. The effective mass m_0 measured in $\beta\beta_{0\nu}$ decay is $|a+2b+2c|$. However, neutrino oscillations constrain $|a+2b+c|$ and $\sqrt{c^2+2d^2}$, as well as $|d/c|$. Using

$$\begin{aligned} |a+2b+2c| &= ||a+2b+c| \pm |c|| \\ &= ||a+2b+c| \pm \cos 2\theta \sqrt{c^2+2d^2}|, \end{aligned} \quad \ldots(14)$$

the following conditions on m_0 are easily obtained:

(1) $m_0 \simeq \sin^2\theta |m_2| \simeq \sin^2\theta \sqrt{\Delta m^2_{sol}}$, $\ldots(15)$

(2) $m_0 \simeq |m_{1,2}| \ll \sqrt{\Delta m^2_{atm}}$, $\ldots(16)$

(3) $m_0 \simeq \cos 2\theta |m_{1,2}| \ll \cos 2\theta \sqrt{\Delta m^2_{atm}}$, $\ldots(17)$

(4) $m_0 \simeq \sqrt{\Delta m^2_{atm}}$, $\ldots(18)$

(5) $m_0 \simeq \cos 2\theta \sqrt{\Delta m^2_{atm}}$, $\ldots(19)$

(6) $m_0 \simeq |m_{1,2,3}|$, $\ldots(20)$

(7) $m_0 \simeq \cos 2\theta |m_{1,2,3}|$. $\ldots(21)$

If m_0 is measured[9] to be significantly larger than 0.05 eV, then only Cases (6) and (7) are allowed. However, as eqs.20 and 21 show, the true mass of the three neutrinos is still subject to a two-fold ambiguity, which is a well-known result.

The underlying symmetry of eq.5 which results in $U_{e3} = 0$ is its invariance under the interchange of ν_μ and ν_τ. Its mass eigenstates are then separated according to whether they are even $(\nu_{1,2})$ or odd (ν_3) under this interchange, as shown by eq.1. To obtain $U_{e3} \neq 0$, this symmetry has to be broken. One interesting possibility is to rewrite eq.5 as

$$\mathcal{M}_\nu = \begin{pmatrix} a+2b+2c & d & d^* \\ d & b & a+b \\ d^* & a+b & b \end{pmatrix}, \ldots(22)$$

where a, b, c are real but d is complex. This reduces to eq.4 if Im $d = 0$, but if Im $d \neq 0$, then $U_{e3} \neq 0$.

To obtain U_{e3} in a general way, first rotate to the basis spanned by $\nu_e, (\nu_\mu+\nu_\tau)/\sqrt{2}$, and $(\nu_\tau - \nu_\mu)/\sqrt{2}$, i.e.,

$$\mathcal{M}_\nu = \begin{pmatrix} a+2b+2c & \sqrt{2}\mathrm{Re}\, d & -\sqrt{2}i\mathrm{Im}\, d \\ \sqrt{2}\mathrm{Re}\, d & a+2b & 0 \\ -\sqrt{2}i\mathrm{Im}\, d & 0 & -a \end{pmatrix} \quad \ldots(23)$$

Whereas $\mathcal{M}_\nu$ is diagonalized by

$$U\mathcal{M}_\nu U^T = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad \ldots(24)$$

$\mathscr{M}_\nu \mathscr{M}_\nu^\dagger$ is diagonalized by

$$U(\mathscr{M}_\nu \mathscr{M}_\nu^\dagger)U^\dagger = \begin{pmatrix} |m_1|^2 & 0 & 0 \\ 0 & |m_2|^2 & 0 \\ 0 & 0 & |m_3|^2 \end{pmatrix}. \qquad \ldots(25)$$

Here

$$\mathscr{M}_\nu \mathscr{M}_\nu^\dagger = \begin{pmatrix} (a+2b+2c)^2+2|d|^2 & 2\sqrt{2}(a+2b+c)\text{Re } d & 2\sqrt{2}i(a+b+c)\text{Im } d \\ 2\sqrt{2}(a+2b+c)\text{Re } d & (a+2b)^2+2(\Re d)^2 & 2i\text{Re } d\text{Im } d \\ -2\sqrt{2}i(a+b+c)\text{Im } d & -2i\text{Re } d\Im d & a^2+2(\text{Im } d)^2 \end{pmatrix} \qquad \ldots(26)$$

To obtain U_{e3} for small Im d, consider the matrix

$$A = \mathscr{M}_\nu \mathscr{M}_\nu^\dagger - [a^2+2(\text{Im } d)^2]I, \qquad \ldots(27)$$

where I is the identity matrix. Now A is diagonalized by U as well and U_{e3} is simply given by

$$U_{e3} \simeq \frac{A_{e3}}{A_{ee}} = \frac{2\sqrt{2}i(a+b+c)\text{Im } d}{(a+2b+2c)^2 - a^2 + 2(\text{Re } d)^2} \ldots(28)$$

to a very good approximation and leads to

$$(1),(2),(3) \qquad U_{e3} \simeq \frac{-\sqrt{2}i\text{Im } d}{a}, \qquad \ldots(29)$$

$$(4),(6) \qquad U_{e3} \simeq \frac{i\text{Im } d}{\sqrt{2}b}, \qquad \ldots(30)$$

$$(5) \qquad U_{e3} \simeq \frac{\sqrt{2}i\text{Im } d}{c}, \qquad \ldots(31)$$

$$(7) \qquad U_{e3} \simeq \frac{\sqrt{2}i(a+c)\text{Im } d}{c^2 - a^2 + 2(\text{Re } d)^2}. \qquad \ldots(32)$$

In all cases, the magnitude of U_{e3} can be as large as the present experimental limit[6] of 0.16 and its phase is $\pm\pi/2$. Thus the CP violating effect in neutrino oscillations is predicted to be maximal by eq.22, which is a very desirable scenario for future long-baseline neutrino experiments.

The above analysis shows that for $U_{e3} = 0$ and $\sin^2 2\theta_{atm} = 1$, the seven cases considered cover all possible patterns of the 3×3 Majorana neutrino mass matrix, as indicated by present atmospheric and solar neutrino data. Any successful model should predict eq.5 at least as a first approximation. One such example already exists[10], where $b = c = d = 0$ corresponds to the non-Abelian discrete symmetry A_4, i.e. the finite group of the rotations of a regular tetrahedron. This leads to Case (6), i.e. three nearly degenerate masses, with the common mass equal to that measured in $\beta\beta_{0\nu}$ decay. It has also been shown recently[11] that starting with this pattern, the correct mass matrix, i.e. eq.22 with the complex phase in the right place, is automatically obtained with the most general application of radiative corrections. In particular, if soft supersymmetry breaking is assumed to be the origin of these radiative corrections, then the neutrino mass matrix is correlated with flavor violation in the slepton sector, and may be tested in future collider experiments.

Suppose that at some high energy scale, the charged lepton mass matrix and the Majorana neutrino mass matrix are such that after diagonalizing the former, i.e.,

$$\mathscr{M}_l = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \qquad \ldots(33)$$

the latter is of the form

$$\mathscr{M}_\nu = \begin{pmatrix} m_0 & 0 & 0 \\ 0 & 0 & m_0 \\ 0 & m_0 & 0 \end{pmatrix}. \qquad \ldots(34)$$

From the high scale to the electroweak scale, one-loop radiative corrections will change $\mathscr{M}_\nu$ as follows:

$$(\mathscr{M}_\nu)_{ij} \to (\mathscr{M}_\nu)_{ij} + R_{ik}(\mathscr{M}_\nu)_{kj} + (\mathscr{M}_\nu)_{ik}R^T_{kj}, \ldots(35)$$

where the radiative correction matrix is assumed to be of the most general form, i.e.,

$$R = \begin{pmatrix} r_{ee} & r_{e\mu} & r_{e\tau} \\ r^*_{e\mu} & r_{\mu\mu} & r_{\mu\tau} \\ r^*_{e\tau} & r^*_{\mu\tau} & r_{\tau\tau} \end{pmatrix}. \qquad \ldots(36)$$

Thus the observed neutrino mass matrix is given by

$$\mathscr{M}_\nu = m_0 \begin{pmatrix} 1+2r_{ee} & r_{e\tau}+r^*_{e\mu} & r_{e\mu}+r^*_{e\tau} \\ r^*_{e\mu}+r_{e\tau} & 2r_{\mu\tau} & 1+r_{\mu\mu}+r_{\tau\tau} \\ r^*_{e\tau}+r_{e\mu} & 1+r_{\mu\mu}+r_{\tau\tau} & 2r^*_{\mu\tau} \end{pmatrix} \ldots(37)$$

Now $r_{\mu\tau}$ may be chosen real by absorbing its phase into ν_μ and ν_τ. Then using the redefinitions:

$$\delta_0 \equiv r_{\mu\mu} + r_{\tau\tau} - 2r_{\mu\tau}, \qquad \ldots(38)$$

$$\delta \equiv 2r_{\mu\tau}, \qquad \ldots(39)$$

$$\delta' \equiv r_{ee} - \frac{1}{2}r_{\mu\mu} - \frac{1}{2}r_{\tau\tau} - r_{\mu\tau}, \qquad \ldots(40)$$

$$\delta'' \equiv r^*_{e\mu} + r_{e\tau}, \qquad \ldots(41)$$

the neutrino mass matrix becomes

$$\mathscr{M}_\nu = m_0 \begin{pmatrix} 1+\delta_0+2\delta+2\delta' & \delta'' & \delta''^* \\ \delta'' & \delta & 1+\delta_0+\delta \\ \delta''^* & 1+\delta_0+\delta & \delta \end{pmatrix}, \qquad \ldots(42)$$

which is exactly that of eq.22. In other words, starting with eq.34, the correct $\mathscr{M}_\nu$ is automatically obtained. [To simplify eq.42 without any loss of generality, δ_0 will be set equal to zero from here on.]

The successful derivation of eq.42 depends on having eqs.33 and 34. To be sensible theoretically, they should be maintained by a symmetry. At first sight, it appears impossible that there can be a symmetry which allows them to coexist. Here is where the non-Abelian discrete symmetry A_4 comes into play[10]. The key is that A_4 has three inequivalent one-dimensional representations $\underline{1}$, $\underline{1}'$, $\underline{1}''$, and one three-dimensional representation $\underline{3}$, with the decomposition

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}. \qquad \ldots(43)$$

This allows the following natural assignments of quarks and leptons:

$$(u_i, d_i)_L, \;\; (\nu_i, e_i)_L \sim \underline{3}, \qquad \ldots(44)$$
$$u_{1R}, \;\; d_{1R}, \;\; e_{1R} \sim \underline{1}, \qquad \ldots(45)$$
$$u_{2R}, \;\; d_{2R}, \;\; e_{2R} \sim \underline{1}', \qquad \ldots(46)$$
$$u_{3R}, \;\; d_{3R}, \;\; e_{3R} \sim \underline{1}''. \qquad \ldots(47)$$

Heavy fermion singlets are then added[11]:

$$U_{iL(R)}, \;\; D_{iL(R)}, \;\; E_{iL(R)}, \;\; N_{iR} \sim \underline{3}, \qquad \ldots(48)$$

together with the usual Higgs doublet and new heavy singlets:

$$(\phi^+, \phi^0) \sim \underline{1}, \quad \chi_i^0 \sim \underline{3}. \qquad \ldots(49)$$

With this structure, charged leptons acquire an effective Yukawa coupling matrix $\overline{e}_{iL} e_{jR} \phi^0$ which has 3 arbitrary eigenvalues (because of the 3 independent couplings to the 3 inequivalent one-dimensional representations) and for the case of equal vacuum expectation values of χ_i, i.e.

$$\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle = u, \qquad \ldots(50)$$

the unitary transformation U_L which diagonalizes $\mathscr{M}_l$ is given by

$$U_L = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \qquad \ldots(51)$$

where $\omega = e^{2\pi i/3}$. This implies that the effective neutrino mass operator, i.e. $\nu_i \nu_j \phi^0 \phi^0$, is proportional to

$$U_L^T U_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \ldots(52)$$

exactly as desired[10, 11].

To derive eq.52, the validity of eq.50 has to be proved. This is naturally accomplished in the context of supersymmetry. Let $\hat{\chi}_i$ be superfields, then its superpotential is given by

$$\hat{W} = \frac{1}{2} M_\chi (\hat{\chi}_1 \hat{\chi}_1 + \hat{\chi}_2 \hat{\chi}_2 + \hat{\chi}_3 \hat{\chi}_3) + h_\chi \hat{\chi}_1 \hat{\chi}_2 \hat{\chi}_3. \qquad \ldots(53)$$

Note that the h_χ term is invariant under A_4, a property not found in $SU(2)$ or $SU(3)$. The resulting scalar potential is

$$V = |M_\chi \chi_1 + h_\chi \chi_2 \chi_3|^2 + |M_\chi \chi_2 + h_\chi \chi_3 \chi_1|^2 + |M_\chi \chi_3 + h_\chi \chi_1 \chi_2|^2. \qquad \ldots(54)$$

Thus a supersymmetric vacuum ($V = 0$) exists for

$$\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle = u = -M_\chi / h_\chi, \qquad \ldots(55)$$

proving eq.50, with the important additional result that the spontaneous breaking of A_4 at the high scale u does not break the supersymmetry.

To generate the proper radiative corrections which will result in a realistic Majorana neutrino mass matrix, A_4 is assumed broken also by the soft supersymmetry breaking terms. In particular, the mass-squared matrix of the left sleptons will be assumed to be arbitrary. This allows $r_{\mu\tau}$ to be nonzero through $\tilde{\mu}_L - \tilde{\tau}_L$ mixing, from which the parameter δ may be evaluated, as shown in Figs.1 and 2. For illustration, using the approximation that $\tilde{m}_1^2 >> \mu^2 >> M_{1,2}^2 = \tilde{m}_2^2$, where μ is the Higgsino mass and $M_{1,2}$ are gaugino masses, I find

$$\delta = \frac{\sin\theta \cos\theta}{16\pi^2} \times \left[(3g_2^2 - g_1^2) \ln \frac{\tilde{m}_1^2}{\mu^2} - \frac{1}{4} (3g_2^2 + g_1^2) \left(\ln \frac{\tilde{m}_1^2}{\tilde{m}_2^2} - \frac{1}{2} \right) \right]. \qquad \ldots(56)$$

Using $\Delta m_{32}^2 = 2.5 \times 10^{-3}$ eV^2 from the atmospheric neutrino data, this implies that

$$\left[\ln \frac{\tilde{m}_1^2}{\mu^2} - 0.3 \left(\ln \frac{\tilde{m}_1^2}{\tilde{m}_2^2} - \frac{1}{2} \right) \right] \sin\theta \cos\theta \simeq 0.535 \left(\frac{0.4 \text{ eV}}{m_0} \right)^2. \qquad \ldots(57)$$

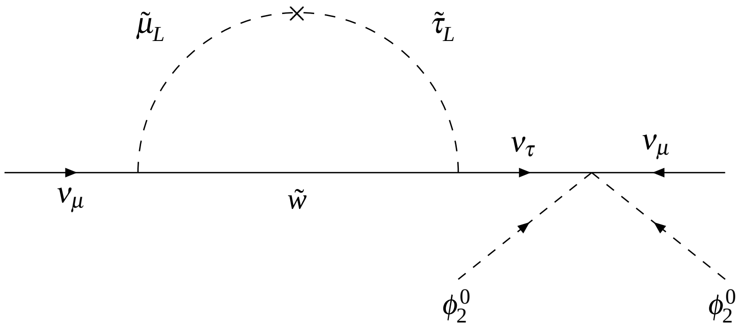

Fig. 1 Wavefunction contribution to δ in supersymmetry.

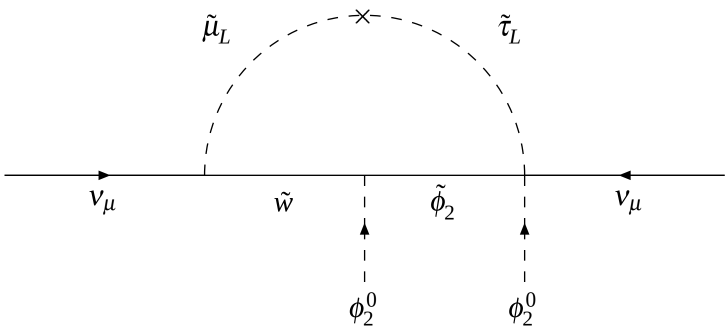

Fig. 2 Vertex contribution to δ in supersymmetry.

To the extent that the factor on the left cannot be much greater than unity, this means that m_0 cannot be much smaller than about 0.4 eV[9].

In the presence of Im δ'', as shown by eq.30,

$$U_{e3} \simeq \frac{i\mathrm{Im}\ \delta''}{\sqrt{2}\delta}, \qquad \ldots(58)$$

and the previous expressions for the neutrino mass eigenvalues are still approximately valid with the replacement of δ' by $\delta' + (\mathrm{Im}\ \delta'')^2/2\delta$ and of δ'' by Re δ''. There is also the relationship

$$\left[\frac{\Delta m^2_{12}}{\Delta m^2_{32}}\right]^2 \simeq \left[\frac{\delta'}{\delta} + |U_{e3}|^2\right]^2 + 2\left[\frac{\mathrm{Re}\ \delta''}{\delta}\right]^2 \ldots(59)$$

Using $\Delta m^2_{12} \simeq 5 \times 10^{-5}$ eV^2 from solar neutrino data and $|U_{e3}| < 0.16$ from reactor neutrino data[6], I find

$$\mathrm{Im}\ \delta'' < 8.8 \times 10^{-4}\ (0.4\ \mathrm{eV}/m_0)^2, \quad \ldots(60)$$
$$\mathrm{Re}\ \delta'' < 5.5 \times 10^{-5}\ (0.4\ \mathrm{eV}/m_0)^2. \quad \ldots(61)$$

In conclusion, recent experimental progress on neutrino oscillations points to a neutrino mixing matrix which can be understood in a systematic way[7] in terms of an all-purpose neutrino mass matrix, i.e. eq.5, and its simple extension, i.e. eq.22, to allow for a nonzero and *imaginary* U_{e3}, i.e. eq.28. Seven possible cases have been identified, each with a different prediction for $\beta\beta_{0\nu}$ decay, i.e. eqs.15 to 21. A specific example is that of an underlying A_4 symmetry at some high energy scale, which allows the observed Majorana neutrino mass matrix to be derived from radiative corrections. It has been shown[11] that this automatically leads to $\sin^2 2\theta_{atm} = 1$ and a large (but not maximal) solar mixing angle. Using neutrino oscillation data, and assuming radiative corrections from soft supersymmetry breaking, the effective mass measured in neutrinoless double beta decay is predicted to be not much less than 0.4 eV.

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