Proc Indian Natn Sci Acad, **70, A** No. 1, January 2004, pp. 123–133

Neutrinos from Supernovae

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(Received 10 January 2003; Accepted 17 July 2003)

In this review, the effect of flavor oscillations on the neutrinos released during supernova explosion after core collapse is described. In some scenarios there is a large enhancement of the number of events compared to the no oscillation case. Various other features associated with supernova neutrinos are also discussed.

Key Words: Core Collapse Supernova; Neutrino Mass and Mixing; Neutrino Detection

1 Introduction

February 23, 1987 saw the birth of a new era in astrophysics-extra-solar system neutrino astronomy. The supernova explosion in the Large Magellanic Cloud (LMC) at a distance of about 50 kpc was not only the closest visual supernova since Kepler but was also the source of neutrinos detected at the terrestrial detectors of Kamiokande (KII) and IMB giving rise to 11 and 8 events respectively.

The next few years saw great excitement in this field. Astrophysics interacted with particle physics intimately. From the number and the energy distribution of the observed neutrinos one tried to extract information about the stellar core and check them with model predictions. On the other hand these neutrinos also gave particle physics constraints on neutrino properties. In the last few years interest in this area got rejuvenated by the finding that neutrinos do have non-zero mass and the flavors do mix when they travel. This conclusion was reached through the analysis of the atmospheric neutrinos detected at the Superkamioka (SK) along with their zenith angle dependence and the observation of the deficit of detected solar neutrinos by the Chlorine and Gallium radiochemical detectors and at SK and Sudbury Neutrino Observatory (SNO) through electron scattering and charged/neutral current dissociation of heavy water respectively. The recent results announced by the KamLAND reactor experiment gives for the first time conclusive evidence for neutrino oscillation using a terrestrial neutrino source and confirms the Large Mixing Angle (LMA) solution to the solar neutrino problem. Thus the present day interest in supernova neutrinos lies around the question: if you have a galactic supernova event today what would be the number of events and their time and energy distributions in the large number of neutrino detectors in operation. The other related question is whether one can get a signature of neutrino oscillation mechanism from the observed data and also how other neutrino properties get constrained. Information about the mechanisms of the supernova explosion is also an area of huge interest.

In this review we survey some of these issues. In section 2 we give a brief overview of the physics of type II supernovae and the emission of neutrinos from them. Section 3 introduces the subject of neutrino oscillation and the impact of vacuum and matter enhanced oscillation on the supernova neutrinos from the core. Section 4 describes the expected number of events in the terrestrial detectors for the different mass and mixing scenarios. Finally section 5 briefly states the other connected issues of supernova neutrino detection.

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2 Type II Supernovae and Neutrino Emission

Stars of masses larger than $8M_\odot$ after burning for millions of years collapse when the nuclear reactions in the core stop with matter consisting mostly of ^{56}Fe like nuclei. This collapse proceeds very fast (timescale of the order of tens of milliseconds) and stops in the central region when its density goes beyond the nuclear matter density with a strong shock starting to travel outward[1]. This shock wave, eventually hitting the outer mantle in a few seconds and supplying the explosion energy of a few times 10^{51} ergs, is believed to be the cause of type II supernova explosion. During this process, the binding energy released comes out almost completely as neutrinos and antineutrinos of three different flavors (e, μ and τ) in the "cooling phase" with the total energy release of the order of 10^{53} ergs. Let us discuss the emission of the neutrinos in some more detail. Firstly during the early stage of the collapse (densities less than 10^{12} g/cc) neutrinos are produced through neutronization

$$e^- + (N,Z) \to (N+1, Z-1) + \nu_e \qquad \ldots(1)$$

$$e^- + p \to n + \nu_e \qquad \ldots(2)$$

where only ν_e (not $\bar{\nu}_e$) are produced. At lower densities these neutrinos have mean free path much larger than the core radius and hence escape. But the total energy of these neutronization neutrinos is much smaller than that in the cooling phase. Even then it is possible to detect them for nearby galactic supernovae at distances within 1 kpc[2]. These neutrinos can give information about the temperature and composition of the core.

The main neutrino emission is during the cooling phase where the thermal $\nu/\bar{\nu}$ are produced through pair production and other processes[3]. Out of these ν_μ, ν_τ, $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$, called collectively as ν_x, interact with matter only through neutral current whereas ν_e and $\bar{\nu}_e$ have both charged current and neutral current interaction with matter. As the matter is neutron-rich the ν_e's interact more with matter than the $\bar{\nu}_e$'s. These neutrinos deep inside the core are in equilibrium with the ambient matter and their energy distributions are close to Fermi-Dirac as seen through simulations and through the analysis of 1987A neutrinos[4]. As the stellar core has a strong density gradient, electron type neutrinos can stay in equilibrium upto larger radius and so the ν_e "neutrinosphere" has the largest radius and smallest temperature. In this article we shall assume that the three types of neutrino gas have Fermi-Dirac distributions with temperatures 11, 16 and 25 MeV for ν_e, $\bar{\nu}_e$ and ν_x respectively.

An important role played by neutrinos in type II supernovae is in the process of "delayed neutrino heating"[5]. In almost all simulations for large mass stars one sees that the shock wave moving outward fast loses energy in dissociating the nuclei in the overlying matter and soon becomes an accretion shock. This shock gets revitalized over the much longer timescale of seconds through the absorption of a small fraction of the thermal neutrinos that radiate out with each neutrino depositing energy of the order of 10 MeV. Large convection in the central regions also helps this process.

3 The Neutrino Oscillation Probabilities

The flavor eigenstate $|\nu_\alpha\rangle$ created inside the supernova can be expressed as a linear superposition of the mass eigenstates such that $|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$, where U is the unitary mixing matrix and the sum is over N neutrino states. After time t, the initial $|\nu_\alpha\rangle$ evolves to $|\nu_\alpha(t)\rangle = \sum_i e^{-iE_i t} U_{\alpha i}|\nu_i\rangle$ where E_i is the energy of the i^{th} mass eigenstate. Then the probability of finding a flavor ν_β in the original ν_α beam after traveling a distance L in vacuum is given by

$$\begin{aligned} P_{\alpha\beta} &= \delta_{\alpha\beta} \\ &-4\sum_{j>i} U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\sin^2\left(1.27\frac{\Delta m^2_{ij}L}{E}\right), \quad \ldots(3) \end{aligned}$$

where $\Delta m^2_{ij} = m_i^2 - m_j^2$ is the mass squared difference.

Over the last few years the idea that neutrinos are not massless but have small masses has become established as a result of Super-Kamiokande (SK) and SNO which have firm evidence for atmospheric and solar neutrino oscillations[6−11]. The SK atmospheric neutrino data demand $\Delta m^2_{32} \sim 3\times10^{-3}$ eV^2 and almost maximal mixing ($\sin^2 2\theta_{23} \approx 1$) [6] while the global solar neutrino data is best explained if $\Delta m^2_{21} \sim 6.1\times 10^{-5}$ eV^2 with large mixing angles ($\tan^2\theta \sim 0.41$)[10, 12]. Very recently the KamLAND reactor antineutrino disappearance experiment [13] provided conclusive confirmation of the LMA solution to the solar neutrino problem, with mass and mixing parameters absolutely consistent with the solar neutrino results. The global analysis of the solar and the KamLAND data[14] gives $\Delta m^2 = 7.17 \times 10^{-5}$ eV^2 and $\tan^2\theta = 0.44$. The CHOOZ reactor experiment restricts $\sin^2\theta_{13} < 0.1$ for $\Delta m^2 > 10^{-3}$ eV^2[15]. The only other positive signal for neutrino oscillations comes from the accelerator experiment LSND which requires $\Delta m^2 \sim$ eV^2 and mixing angle small ($\sin^2 2\theta \sim 10^{-3}$). To include LSND in the framework of oscillation one needs to extend the number of neutrino generations to four, or in other words, include a sterile neutrino. However with the latest SNO data on solar neutrinos and the final data from SK on atmospheric neutrinos, both the "2+2" and "3+1" 4-generation scenarios fail to explain the global neutrino data. While "3+1" is inadequate in explaining the combined accelerator-reactor data including LSND, "2+2" cannot accommodate the solar and atmospheric neutrino data together[16].

Since galactic supernova neutrinos with energies ~ 10 MeV travel distances ~ 10 Kpc ($\sim 3 \times 10^{20}$ m), the coherent term in eq.3 becomes important only for $\Delta m^2_{ij} \sim 10^{-19}$ eV^2. Thus supernova neutrinos can be used as probes for mass squared differences, not possible to detect with any known terrestrial source. However for the solar and atmospheric mass scales given above the oscillatory term would average out to 1/2.

The expression (3) would have been correct/exact if the neutrinos were traveling in vacuum. However for the supernova neutrinos things are a little complicated since they are created deep inside the core and traverse through extremely dense matter before they come out into the vacuum. As the neutrinos move in matter they undergo scattering with the ambient electrons. While all the active neutrino flavors scatter electrons by the neutral current process, only the ν_e (and $\bar{\nu}_e$) have charged current interactions as well. This significantly affects neutrino oscillations parameters as the ν_e picks up an additional matter induced mass term[17]

$$A(r) = 2\sqrt{2} G_F N_A n_e(r) E \qquad \ldots(4)$$

where G_F is the Fermi constant, N_A the Avogadro's number, $n_e(r)$ the ambient electron density in the supernova at radius r and E the energy of the neutrino beam. In appropriate units

$$\frac{A(r)}{\mathrm{eV}^2} = 15.14 \times 10^{-8} Y_e(r) \frac{\rho(r)}{\mathrm{gm/cm}^3} \frac{E}{\mathrm{MeV}} \qquad \ldots(5)$$

where $Y_e(r)$ is the electron fraction and $\rho(r)$ is the matter density profile in the supernova which can be very well approximated by a power law $\rho(r) = Cr^{-n}$ with $n = 3$ in the core[18].

Neutrino oscillation probabilities may also be significantly affected inside Earth as the neutrinos traverse the Earth matter[19]. Thus the neutrino oscillation probability is given by

$$P_{\alpha\beta} = \sum_{i=1}^{N} P^m_{\alpha i} P^{\oplus}_{i\beta}, \qquad \ldots(6)$$

where

$$P^m_{\alpha i} = \sum_{j=1}^{N} \left|U^m_{\alpha j}\right|^2 \left|\langle \nu_i | \nu^m_j \rangle\right|^2 \qquad \ldots(7)$$

is the probability that a ν_α ($\alpha = e, \mu, \tau$) produced inside the supernova core would emerge as a ν_i ($i = 1,2,3$) at the surface of the supernova, $U^m_{\alpha j}$ are the elements of the mixing matrix at the point of production and $|\langle \nu_i | \nu^m_j \rangle|^2$ is the probability that a $|\nu^m_j\rangle$ state in matter appears as the state $|\nu_i\rangle$ at the supernova surface in vacuum. This is the so called "jump probability". $P^{\oplus}_{i\beta}$ is the probability that the ν_i mass eigenstate arriving at the surface of the Earth is detected as a ν_β flavor state in the detector. Depending on whether the neutrinos cross the Earth or not, $P^{\oplus}_{i\beta}$ maybe different from $|U_{\beta i}|^2$, where $U_{\beta i}$ is the element of the mixing matrix in vacuum.

Since to a good approximation the average energy and the total fluxes of ν_μ, $\bar{\nu}_\mu$, ν_τ and $\bar{\nu}_\tau$ are same, for mixing between only active neutrino flavors the only relevant oscillation probability that we need is the ν_e survival probability P_{ee} which is given by eq.6 with $\alpha = \beta = e$.

Two Flavor Oscillations

To begin with let us for simplicity assume that there are just two neutrino flavors, ν_e and another active flavor ν_a which may be ν_μ or ν_τ. The effective mixing angle in matter is given by

$$\tan 2\theta_m(r) = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A(r)}. \qquad \ldots(8)$$

Since the density inside the supernova core where the neutrinos are created is extremely high, $A(r_s) \gg \Delta m^2$ and $\theta_m \approx \pi/2$. Hence the survival probability P_{ee} given by eqs.6 and 7 reduces to

$$P_{ee} = P_J P^{\oplus}_{1e} + (1 - P_J) P^{\oplus}_{2e}, \qquad \ldots(9)$$

where $P_J = |\langle \nu_1 | \nu_2^m \rangle|^2$ is the jump or the crossing probability from one neutrino mass eigenstate to the other at resonance. If $P_J \approx 0$ the neutrino propagation in matter is called *adiabatic*, otherwise it is *non-adiabatic*. When $P_J \to 1$ then we encounter the extreme non-adiabatic situation. In ref.[20] it is shown that the *double-exponential* parametrization of P_J derived in ref.[21] and used extensively for the solar neutrinos, works extremely well even for the supernova density profile. In this parametrization the jump probability is expressed as

$$P_J = \frac{\exp(-\gamma \sin^2\theta) - \exp(-\gamma)}{1 - \exp(-\gamma)}, \qquad \ldots(10)$$

where γ is given by

$$\gamma = \pi \frac{\Delta m^2}{E} \left| \frac{dln\, n_e}{dr} \right|^{-1}_{r = r_{mva}}. \qquad \ldots(11)$$

The *density scale factor* $\left| \frac{dln\, n_e}{dr} \right|^{-1}$ gives a measure of the deviation from adiabaticity and is calculated at the position where we have *maximum violation of adiabaticity (mva)*[20, 22]. That is where

$$A(r_{mva}) = \Delta m^2 . \qquad \ldots(12)$$

Note that the position of *mva* (r_{mva}) is different from the position of resonance (r_{res}) which is given by the condition

$$A(r_{res}) = \Delta m^2 \cos 2\theta. \qquad \ldots(13)$$

The form of the probability $P_{ie}^{\oplus}$ depends crucially on the trajectory of the neutrinos inside the earth and hence on the direction of the supernova. If the direction is such that the neutrinos cross only the mantle of the Earth then the amplitude is given by

$$A_{2e}^{\oplus} \;=\; \sum_j U_{ej}^e e^{-i\phi_j^e} \langle \nu_j^e | \nu_i \rangle, \qquad \ldots(14)$$

where U_{ej}^e is the mixing matrix elements in the Earth's mantle and ϕ_j^e is the phase. Therefore the expression for $P_{2e}^{\oplus} (= 1 - P_{1e}^{\oplus})$ is given by

$$P_{2e}^{\oplus} = \sin^2\theta$$
$$+ \sin 2\theta_e \sin(2\theta_e - 2\theta) \sin^2 \left(1.27 \frac{\Delta m_e L}{E} \right), \ldots(15)$$

where L is the distance traversed inside Earth and θ_e (given by eq.8) but with A calculated in the mantle of the Earth) and Δm_e^2 are the mixing angle and the mass squared difference inside the Earth's mantle. If the neutrinos cross both the mantle as well as the core of the Earth then

$$A_{2e}^{\oplus} \;=\; \sum_{\substack{i,j,k,\\ \alpha,\beta,\sigma}} U_{ek}^M e^{-i\psi_k^M} U_{\alpha k}^M U_{\alpha i}^C e^{-i\psi_i^C}$$
$$U_{\beta i}^C U_{\beta j}^M e^{-i\psi_j^M} U_{\sigma j}^M U_{\sigma 2}, \qquad \ldots(16)$$

where (i,j,k) denotes mass eigenstates and (α, β, σ) denotes flavor eigenstates, U, U^M and U^C are the mixing matrices in vacuum, in the mantle and the core respectively and ψ^M and ψ^C are the corresponding phases picked up by the neutrinos as they travel through the mantle and the core of the Earth. Then the probability is given by

$$P_{2e}^{\oplus} = |A_{2e}^{\oplus}|^2. \qquad \ldots(17)$$

The additional mass term picked up by the $\bar{\nu}_e$ as it moves in matter is $-A(r)$. Since the crucial combination which decides matter effects is the ratio $A(r)/\Delta m^2$, the antineutrino survival probability $\bar{P}_{ee}$ is identical to that for the neutrinos if we change the sign of Δm^2, which is equivalent to swapping of the mass labels $1 \leftrightarrow 2$[20]. Then the expression for $\bar{P}_{ee}$ is similar to that for P_{ee} and is given by

$$\bar{P}_{ee} = \bar{P}_J \bar{P}_{1e}^{\oplus} + (1 - \bar{P}_J) \bar{P}_{2e}^{\oplus}, \qquad \ldots(18)$$

where

$$\bar{P}_J = \frac{\exp(-\gamma \cos^2\theta) - \exp(-\gamma)}{1 - \exp(-\gamma)}, \qquad \ldots(19)$$

where we replace $\cos^2\theta$ with $\sin^2\theta$ (swapping $1 \leftrightarrow 2$) and γ is calculated at r_{mva} given by the same eq.12. The expressions for the oscillation probabilities $\bar{P}_{2e}$ are again similar to those for the neutrinos

$$\bar{P}_{2e}^{\oplus} = \sin^2\theta$$
$$+ \sin 2\bar{\theta}_e \sin(2\bar{\theta}_e - 2\theta) \sin^2 \left(1.27 \frac{\Delta \bar{m}_e L}{E} \right), \ldots(20)$$

where eq.20 is for transition probability in Earth for one slab approximation, with the mixing angle $\bar{\theta}_e$ given by

$$\tan 2\bar{\theta}_e = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta + A(r)}. \qquad \ldots(21)$$

The expression for $\bar{P}_{2e}^{\oplus}$ for two slabs can also be similarly derived from eq.17 with the corresponding changes for the antineutrinos.

Three Flavor Oscillations

We now consider a more realistic scenario with mixing between three active neutrinos, with one of the mass squared differences corresponding to the solar scale ($\Delta m_{21}^2 \sim 10^{-5}$ eV^2) and the other one corresponding to the atmospheric scale ($\Delta m_{31}^2 \sim 10^{-3}$ eV^2). In this case the neutrinos encounter two resonances, the first one corresponding to the higher scale at a higher density in the supernova and the next one corresponding to the lower mass scale further out in the mantle. Though from solar neutrino data we know that the sign of $\Delta m_{21}^2 \equiv \Delta m_{\odot}^2$ is positive[10] ($\Delta m_{ij}^2 = m_i^2 - m_j^2$), there is still an ambiguity in the sign of $\Delta m_{32}^2 \equiv \Delta m_{atm}^2$. It would be hard to determine the sign of Δm_{32}^2 in any of the current and planned long baseline oscillation experiments and only a neutrino factory would be able to resolve this ambiguity. However for the supernova neutrinos the sign of $\Delta m_{32}^2 \sim \Delta m_{31}^2$ is crucial and thus the supernova neutrinos can be used very effectively to give us the neutrino mass hierarchy.

Direct Mass Hierarchy Since the density at the neutrino source (r_s) is very high, $A(r_s) \gg \Delta m_{31}^2 \gg \Delta m_{21}^2$ and we can solve the eigenvalue problem perturbatively to get the mixing angles for neutrinos[20, 23, 24, 25] in matter[b]

$$\tan 2\theta_{12}^m(r) = \frac{\Delta m_{21}^2 \sin 2\theta_{12}}{\Delta m_{21}^2 \cos 2\theta_{12} - \cos^2\theta_{13} A(r)} \quad \ldots(22)$$

$$\tan 2\theta_{13}^m(r) = \frac{\Delta m_{31}^2 \sin 2\theta_{13}}{\Delta m_{31}^2 \cos 2\theta_{13} - A(r)} \quad \ldots(23)$$

At the point of production since $A(r_s) \gg \Delta m_{31}^2 \gg \Delta m_{21}^2$ from eqs.22 and 23 we see that $\theta_{12}^m \approx \pi/2 \approx \theta_{13}^m$ and neutrinos are created in almost pure ν_3^m states and the expression for the survival probability for this three-generation scenario is

$$P_{ee} = P_H P_L P_{1e}^{\oplus} + P_H(1-P_L)P_{2e}^{\oplus} + (1-P_H)P_{3e}^{\oplus}, \ldots(24)$$

where P_H and P_L are the jump probabilities for the high and low density transitions respectively. Just like in the two-generation case they can be calculated using the double exponential forms with

$$P_L = \frac{\exp(-\gamma_L \sin^2\theta_{12}) - \exp(-\gamma_L)}{1-\exp(-\gamma_L)}, \quad \ldots(25)$$

$$P_H = \frac{\exp(-\gamma_H \sin^2\theta_{13}) - \exp(-\gamma_H)}{1-\exp(-\gamma_H)}, \ldots(26)$$

where $\gamma_{L,H}$ is calculated using eq.11 at the position of maximum violation of adiabaticity corresponding to the lower (r_L) and the higher scales (r_H) respectively given by the relations

$$\cos^2\theta_{13} A(r_L) = \Delta m_{21}^2, \quad \ldots(27)$$

$$A(r_H) = \Delta m_{31}^2. \quad \ldots(28)$$

For the antineutrinos $\bar{\nu}_e$ the matter term is negative so as in the two-generation case the mixing angle for the antineutrinos in matter is given by

$$\tan 2\bar{\theta}_{12}^m(r) = \frac{\Delta m_{21}^2 \sin 2\theta_{12}}{\Delta m_{21}^2 \cos 2\theta_{12} + \cos^2\theta_{13} A(r)} \quad \ldots(29)$$

$$\tan 2\bar{\theta}_{13}^m(r) = \frac{\Delta m_{31}^2 \sin 2\theta_{13}}{\Delta m_{31}^2 \cos 2\theta_{13} + A(r)} \quad \ldots(30)$$

which implies that at the point of production $\cos 2\bar{\theta}_{12}^m \approx +1 \approx \cos 2\bar{\theta}_{13}^m$ ($\bar{\theta}_{12}^m \approx 0 \approx \bar{\theta}_{13}^m$) and the antineutrinos are created in pure $\bar{\nu}_1^m$ state. Thus for the antineutrinos the survival probability is given by[20]

$$\bar{P}_{ee} = (1-\bar{P}_L)\bar{P}_{1e}^{\oplus} + \bar{P}_L \bar{P}_{2e}^{\oplus}, \quad \ldots(31)$$

where the jump probability $\bar{P}_L$ for the antineutrinos is given by

$$\bar{P}_L = \frac{\exp(-\gamma_L \cos^2\theta_{12}) - \exp(-\gamma_L)}{1-\exp(-\gamma_L)} \quad \ldots(32)$$

with γ_L defined by eq.27 and eq.11.

Inverse Mass Hierarchy If $\Delta m_{31}^2 \approx \Delta m_{32}^2$ is negative, the mixing angles for the neutrinos are still given by the eqs.22 and 23 but with the sign of Δm_{31}^2 reversed.[c] At the production point then $\theta_{12}^m \approx \pi/2$ while $\theta_{13}^m \approx 0$ and ν_e's are thus in almost pure ν_2^m states and the neutrino survival probability is

$$P_{ee} = P_L P_{1e}^{\oplus} + (1-P_L)P_{2e}^{\oplus} \quad \ldots(33)$$

with the jump probability P_L given by eqs.25, 27 and 11.

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[b] If we choose the standard parametrization of the mixing matrix, the mixing angle θ_{23} does not affect the ν_e survival probability and thus we can either choose to rotate it away or even put it to zero without loss of generality.

[c] Note that we take the sign of Δm_{21}^2 as +ve in accordance with currently favored LMA MSW solutions to the solar neutrino problem[12].

With inverse hierarchy the antineutrino mixing angles are given by eqs.29 and 30 with the sign of Δm^2_{31} reversed. Therefore $\bar{\nu}_e$ are created in pure $\bar{\nu}^m_3$ states and their survival probability is[20]

$$\bar{P}_{ee} = (1-\bar{P}_L)P_H\bar{P}^{\oplus}_{1e} + \bar{P}_L P_H \bar{P}^{\oplus}_{2e} + (1-P_H)\bar{P}^{\oplus}_{3e} \quad ..(34)$$

with $\bar{P}_L$ given by eq.32 and P_H by eq.26.

4 Event Rates in Terrestrial Detectors

Neutrinos are created deep inside the supernova core as $\nu - \bar{\nu}$ pairs. They stream out through the supernova core, mantle and envelope and reach the Earth after traveling distances $\sim 10^{17}$ km. In the presence of neutrino oscillations there is a modification of the neutrino fluxes as they oscillate into one another and the resultant neutrino beam at Earth is given by

$$\begin{aligned} N_{\nu_e} &= P_{ee}(E)N^0_{\nu_e}(t) + P_{\mu e}(E)N^0_{\nu_\mu}(t) + P_{\tau e}(E)N^0_{\nu_\tau}(t) \\ &= P_{ee}(E)N^0_{\nu_e}(t) + (1-P_{ee}(E))N^0_{\nu_x}(t) \quad \ldots(35) \\ N_{\bar{\nu}_e} &= \bar{P}_{ee}(E)N^0_{\bar{\nu}_e}(t) + (1-\bar{P}_{ee}(E))N^0_{\nu_x}(t) \quad \ldots(36) \\ N_{\nu_x} &= \frac{1}{2}(1-P_{ee}(E))N^0_{\nu_e}(t) + \frac{1}{2}(1+P_{ee}(E))N^0_{\nu_x}(t) \\ &\qquad \ldots(37) \\ N_{\bar{\nu}_x} &= \frac{1}{2}(1-\bar{P}_{ee}(E))N^0_{\bar{\nu}_e}(t) + \frac{1}{2}(1+\bar{P}_{ee}(E))N^0_{\nu_x}(t) \\ &\qquad \ldots(38) \end{aligned}$$

where P_{ee} and $\bar{P}_{ee}$ are the ν_e and $\bar{\nu}_e$ survival probabilities given in the previous section and $N^0_{\nu_\alpha}(t)$ is the neutrino flux produced inside the supernova core given by $N^0_{\nu_\alpha}(t) = L_{\nu_\alpha}(t)/\langle E_{\nu_\alpha}(t)\rangle$, where $L_{\nu_\alpha}(t)$ is the neutrino luminosity and $\langle E_{\nu_\alpha}(t)\rangle$ is the average energy. In the above expressions we have used the fact that the $\nu_\mu/\bar{\nu}_\mu$ beam is indistinguishable from the $\nu_\tau/\bar{\nu}_\tau$ beam in flux and energy and call them ν_x.

The current and planned terrestrial detectors are capable of observing the supernova neutrinos through various charged and neutral current processes. The differential number of neutrino events at the detector for a given reaction process is

$$\frac{d^2S}{dE_\nu dt} = \sum_\alpha \frac{n}{4\pi D^2} N_{\nu_\alpha} f_{\nu_\alpha}(E_\nu)\sigma(E_\nu)\varepsilon(E_\nu), \quad ..(39)$$

where α runs over the neutrino species concerned (e, μ, τ), N_{ν_α} is the neutrino flux *at the detector* given by eqs. 35)–(38 and $\sigma(E_\nu)$ is the reaction cross-section for the neutrino with the target particle, D is the distance of the neutrino source from the detector (taken as 10kpc for galactic supernovae considered here), n is the number of detector particles for the reaction considered and $f_{\nu_\alpha}(E_\nu)$ is the energy spectrum for the neutrino species involved, while $\varepsilon(E_\nu)$ is the detector efficiency as a function of the neutrino energy.

The main reaction process by which the water Čerenkov detectors like SK would observe the supernova neutrinos is

$$\bar{\nu}_e + p \rightarrow e^+ + n \quad \ldots(40)$$

However the other neutrino species are also observable in SK through the $\nu - e$ elastic scattering processes

$$\nu_i + e^- \rightarrow \nu_i + e^- \quad \ldots(41)$$

In addition to the above two reactions, supernova neutrinos can also be traced in the water Čerenkov detectors through reactions involving ^{16}O. The oxygen nuclei in water are doubly closed shell and have a very high threshold (E_{th}) for excitation. Thus solar neutrinos are unable to have charged or neutral current reactions on oxygen. But supernova neutrinos with much larger energy range can trigger charged current reactions[26]

$$\begin{aligned} \nu_e + {}^{16}O &\rightarrow e^- + {}^{16}F \quad \ldots(42) \\ \bar{\nu}_e + {}^{16}O &\rightarrow e^+ + {}^{16}N \quad \ldots(43) \end{aligned}$$

and neutral current reaction[27]

$$\nu_x + {}^{16}O \rightarrow \nu_x + {}^{16}O^* \quad \ldots(44)$$

where $^{16}O^*$ decays by n, p or γ emission. The reaction thresholds for the charged current reactions (42) and (43) are 15.4 MeV and 11.4 MeV respectively[26]. The electrons from the charged current reactions on ^{16}O can be distinguished in principle from the positrons from $\bar{\nu}_e$ capture on protons (cf. reaction (40)) by their angular distribution. While the ^{16}O events are backward peaked and electron scattering events are strongly forward peaked, the $\bar{\nu}_e p$ events are mostly isotropic. Thus even though all these processes are detected via the Čerenkov , it is possible to disentangle them.

In heavy water (D_2O) detectors like SNO, in addition to the reactions involving elastic scattering off electrons and reactions on ^{16}O, neutrinos can be observed by the charged and neutral current breakup of deuteron

$$\nu_e + d \rightarrow p + p + e^- \qquad \ldots(45)$$

$$\bar{\nu}_e + d \rightarrow n + n + e^+ \qquad \ldots(46)$$

$$\nu_x + d \rightarrow p + n + \nu_x \qquad \ldots(47)$$

The charged current reactions are detected by the Čerenkov radiation from the electron/positron. The neutral current reaction, which will give us information about the total neutrino flux from the supernova, irrespective of whether they oscillate or not, is detected by the capture of the released neutron, either on deuteron or on ^{35}Cl (salt). In the last phase of SNO the neutral current process will be detected by directly observing the neutrons in helium proportional counters.

There have been various attempts before to estimate the effect of non-zero neutrino mass and mixing on the expected neutrino signal from a galactic supernova. With vacuum oscillations we can expect an increase in both the ν_e and $\bar{\nu}_e$ signal[28, 29]. Some special cases where the matter effects inside the supernova are negligible and one has almost pure vacuum oscillations have been considered in ref.[29]. However for the currently most preferred neutrino mass spectrum one expects to have substantial matter effects. Matter enhanced resonant flavor conversion has been observed to have a large effect on the ν_e signal[28,30–35].

Table I gives the calculated number of events expected from the main reactions in H_2O and D_2O, for a typical galactic supernova with a total luminosity of about 3×10^{53} ergs. The numbers here correspond to a three-flavor oscillation scenario with complete flavor conversion. The θ_{13} considered here is *large* so that both P_L and P_H are almost zero, the propagation is almost adiabatic and hence $P_{ee} \approx 0$. The θ_{12} considered is very small and hence $\bar{P}_{ee} \approx 1$[d]. For the cross-section of the $(\nu_e - d), (\bar{\nu}_e - d), (\nu_x - d)$ and $(\bar{\nu}_e - p)$ reactions we refer to Burrows[3]. The cross-section of the $(\nu_e(\bar{\nu}_e) - e^-)$ and $(\nu_x - e^-)$ scattering has been taken from ref.[36] while the neutral current $(\nu_x - {}^{16}O)$ scattering cross-section is taken from ref.[37]. For the $^{16}O(\nu_e, e^-)^{16}F$ and $^{16}O(\bar{\nu}_e, e^+)^{16}N$ reactions we refer to [26] where we have used the cross-sections for the detector with perfect efficiency.

Table I

[Signal from a galactic supernova for complete conversion] The expected number of neutrino events in SNO. To get the number of events in SK, one has to scale the number of events in H_2O *given here to its fiducial mass of 32 kton. The column A corresponds to massless neutrinos, column B to neutrinos with complete flavor conversion* ($P_{ee} \approx 0$)*. The mixing angle* θ_{12} *is considered to be very small corresponding to the SMA solution and hence* $\bar{P}_{ee} \approx 1$*. The* ν_i *here refers to all the six neutrino species.*

		A	B
reactions in 1 kton D_2O	$\nu_e + d \rightarrow p + p + e^-$	75	239
	$\bar{\nu}_e + d \rightarrow n + n + e^+$	91	91
	$\nu_i + d \rightarrow n + p + \nu_i$	544	544
	$\nu_e + e^- \rightarrow \nu_e + e^-$	4	6
	$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$	1	1
	$\nu_{\mu,\tau}(\bar{\nu}_{\mu,\tau}) + e^- \rightarrow \nu_{\mu,\tau}(\bar{\nu}_{\mu,\tau}) + e^-$	4	3
	$\nu_e + {}^{16}O \rightarrow e^- + {}^{16}F$	1	55
	$\bar{\nu}_e + {}^{16}O \rightarrow e^+ + {}^{16}N$	4	4
	$\nu_i + {}^{16}O \rightarrow \nu_i + \gamma + X$	21	21
reactions in 1.4 kton H_2O	$\bar{\nu}_e + p \rightarrow n + e^+$	357	357
	$\nu_e + e^- \rightarrow \nu_e + e^-$	6	9
	$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$	2	2
	$\nu_{\mu,\tau}(\bar{\nu}_{\mu,\tau}) + e^- \rightarrow \nu_{\mu,\tau}(\bar{\nu}_{\mu,\tau}) + e^-$	6	5
	$\nu_e + {}^{16}O \rightarrow e^- + {}^{16}F$	2	86
	$\bar{\nu}_e + {}^{16}O \rightarrow e^+ + {}^{16}N$	6	6
	$\nu_i + {}^{16}O \rightarrow \nu_i + \gamma + X$	33	33

Hence the ν_e flux though depleted in number, gets enriched in high energy neutrinos and since the detection cross-sections are strongly energy dependent, this results in the enhancement of the charged current signal[29]. Since the cross-section for the ^{16}O reactions have the strongest dependence on energy, they are most affected by neutrino oscillations and can be used as an effective way to study neutrino properties from supernova neutrino detection. For the neutral current sector the number of events remain unchanged as the interaction is flavor blind.

From a comparison of the predicted numbers in Table I it is evident that neutrino oscillations play a significant role in supernova neutrino detection. As the average energy of the ν_μ/ν_τ is greater than the average energy of the ν_e, neutrino flavor mixing modifies their energy spectrum. Figure 1 taken from ref.[33], shows the comparison between the total charged current events as a function of the electron/positron energy observed in H_2O ($\bar{\nu}_e p$ events) and D_2O (sum of $\nu_e d$ and $\bar{\nu}_e d$ events) for small and large values of the mixing angle $\sin^2 2\theta_{12}$ ($\omega \equiv \theta_{12}$). The value of $\sin^2\theta_{13} \equiv \varepsilon$ is large ($= 0.08$) which implies that the neutrino propagation is fully adiabatic. Figure 2 also taken from ref.[33], shows the

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[d] The Table I is just for the purpose of illustration only. For the LMA solution the ν_e events would still remain the same, while the $\bar{\nu}_e$ events would be slightly enhanced.

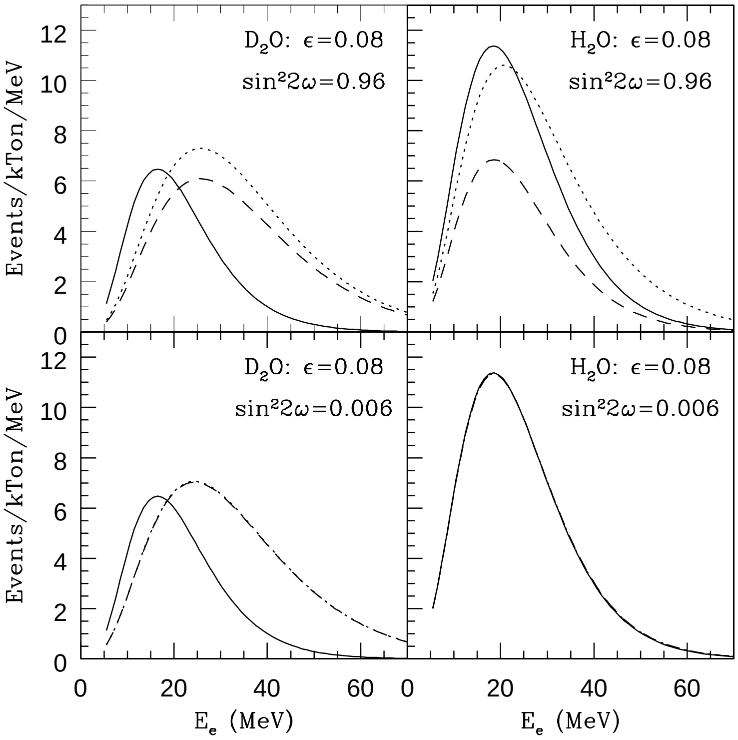

Fig. 1 Total event rates (from combining the individual $\nu_e d$ and $\overline{\nu}_e d$) are shown as a function of the electron/positron energy, E_e, for two different values of $\omega \equiv \theta_{12}$, and for $\epsilon \equiv \sin^2\theta_{13} = 0.08$ so that the propagation is fully adiabatic. The solid lines represent the no mixing case. The dotted and dashed lines are due to the effects of 3- and 4-flavour mixing. Results from a 1 kton water detector (from $\overline{\nu}_e p$ alone) are shown on the right, for comparison. This figure is taken from ref.[33].

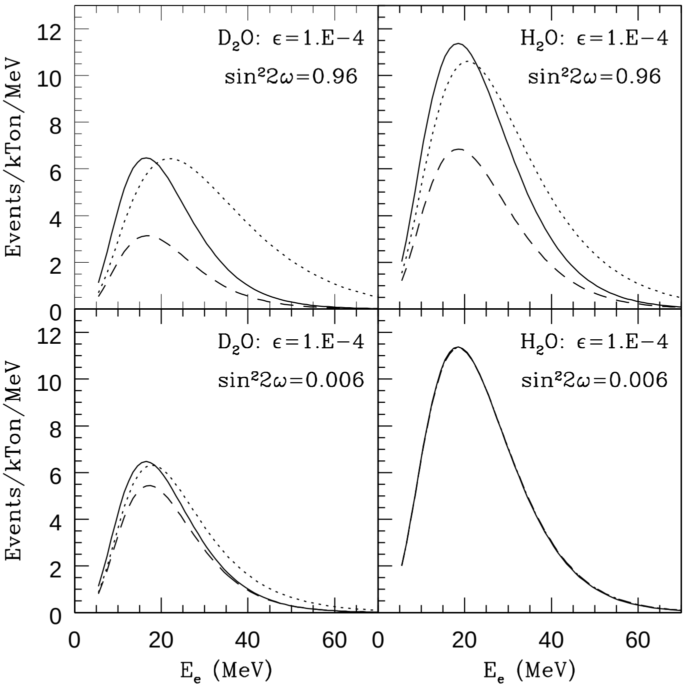

Fig. 2 Same as Fig. 1 but for $\epsilon \sim 0$ so that non-adiabatic effects are included. This figure is taken from ref.[33].

corresponding plots when $\sin^2\theta_{13} \equiv \epsilon$ is small (~ 0), which implies that the neutrino propagation is non-adiabatic. In both the figures the solid lines give the no oscillation distribution and the dotted line the events for three-generation scenario, while the dashed lines correspond to the distribution for a four-generation scheme[e]. The figures show that for the LMA solution (upper panels) there is a shift in the spectral peak in D_2O for both three as well as four generations, the shift being more pronounced for the high ϵ (adiabatic) case. The corresponding shifts in H_2O are less. The SMA cases shown in the lower panels are of less interest now since SMA solution is now ruled out. The figures show that by comparing the signal in SK and SNO one can distinguish between the three and four-generation scenario. But again the four-generation schemes are largely disfavored by the global solar and atmospheric neutrino data[16].

The potential for detecting supernova neutrinos in scintillation detectors like Borexino[38] and MiniBOONE[39] have been recently considered. The ^{12}C in these detectors can be excited through charged current and neutral current interactions with the supernova neutrinos. The charged current reaction ($^{12}C(\nu_e, e^-)^{12}N$) has a threshold of 17.34 MeV while that for ($^{12}C(\bar{\nu}_e, e^+)^{12}B$) has a threshold of 14.4 MeV. But here again the cross-sections have a strong energy dependence and these events show a dramatic increase with large conversion with oscillations compared to the no oscillation case. The neutral current events through ($^{12}C(\nu_x, \nu_x)^{12}C^*$) can be used to put direct limits on the neutrino masses using the time delay techniques briefly discussed in the following section.

5 Other Effects of Neutrino Mass and Mixing

In this section we briefly touch upon a number of areas where the mass and mixing of supernova neutrinos can lead to interesting effects:

a) **SN 1987A**: The eleven SN 1987A events at KII were observed within a timespan of 5.6 secs and with an energy range of the positron/electron released in the water Cerenkov detector from 7.5 MeV to 35.4 MeV. Similarly IMB had the eight events within a time of 12.4 secs and with the electron energy range of 20 MeV to 40 MeV[4]. The angle of the e^+/e^- path to the $\nu/\bar{\nu}$ direction for each event was also measured. There were also 5 events at Mt. Blanc and 3 in Baksan at the same time[4]. A number of analyses were done in the next few years and the results more-or-less agreed with the typical values given in section 2 for the luminosities, average energies and spectra of the neutrinos, though the IMB events gave average energy and temperature consistently higher[40, 41].

Also with such small samples there were large errors in the extraction of the SN parameters. However even though the statistics were poor the SN1987A data was used extensively to study the neutrino mass and mixing patterns. In the context of two flavors such analysis was done by Smirnov et al[42] and Jegerlehner et al[43] and recently it was extended to three flavors in[44]. The authors of ref.[44] claim that the inverse mass hierarchy is disfavored by the data unless θ_{13} is very small, $\sin^2\theta_{13} < 10^{-4}$. However the authors of ref.[45]dispute this observation and conclude that the SN1987A data cannot distinguish between the direct and inverted mass hierarchies. In ref.[46]the SN1987A data is combined with the global solar neutrino data and it is found that while all the other large mixing angle solutions (LOW-QVO and VO) are disfavored, the LMA solution remains the only allowed solution which can explain the SN1987A and the solar neutrino observations simultaneously. Nowadays after the evidence of neutrino mass and mixing one has to work on the "inverse problem" using SN 1987A data to extract the original neutrino spectra using realistic (Large Mixing Angle solution) scenario of neutrino oscillation[47, 48].

b) **Detection of Neutronisation Neutrinos**: The neutrinos emitted during the collapse phase due to the neutronisation give rise to a luminosity small compared to the thermal post-bounce neutrinos discussed above, but for close enough (1 kpc) galactic supernovae they can still be detected by SK and SNO[2]. The measurement of the fluence of these neutrinos at SNO and the distortion of the spectrum detected at SK, in particular the ratio of the calorimetric detection of the neutrino fluence via the neutral current channel to the total energy integrated fluence observed via the charged current channel at SNO can yield valuable information about the mass squared difference and mixing[49].

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[e] We have not considered the four-generation scenario in this review. For a detailed discussion on the four-generation neutrino mass spectrum and its effects on supernova neutrino detection refer to[34, 33].

c) **Delay of Massive Supernova Neutrinos**: For a neutrino of mass m (in eV) and energy E (in MeV) the delay (in sec) in traveling a distance (in 10 kpc) is

$$\Delta t(E) = 0.515(m/E)^2 D \qquad \ldots(48)$$

neglecting small higher order terms. If we assume that the mass of the ν_x is much larger than those of ν_e and $\bar{\nu}_e$ then the neutral current events will have a delay compared to the charged current events. This difference due to time-of-flight for neutral current signal compared to the charged current signal in SNO can determine ν_μ and ν_τ mass down to 30 eV, an improvement by many orders of magnitude over current estimates[37]. One also sees that one can construct[50] useful diagnostic tools for neutrino mass and mixing using the charged and neutral current events as a function of time but only for mass squared differences of the order of tens of eV^2.

d) **Effect of Neutrino Mixing on Delayed Neutrino Heating**: To generate a stronger shock in the supernova models one thinks of mechanisms of extra heating in the region near the shock. As the heating rate due to neutrino capture depends on the square of the neutrino temperature, if the ν_μ or ν_τ emitted from the neutrino sphere can get converted to ν_e before reaching the shockfront, it heats up the shock more. Fuller et al[51] in their numerical calculations got 60% more heating but with the ν/τ neutrino mass of 40 eV. However with realistic solar and atmospheric mass squared differences one does not get this conversion to ν_e inside the stalled shock. Recently it is proposed that the neutrino signal in present and future neutrino detectors can give valuable information about the mechanism of shock propagation and the delayed neutrino heating[52]. When the shock front moves through the MSW conversion region the μ, τ to e type neutrino conversion gets stopped during that time leading to a detectable dip in the neutrino energy/count rate.

e) **r-process nucleosynthesis**: The neutrino-driven-wind environment in the late time (about 3–15 secs after bounce) of core collapse supernova is considered to be a very promising site for the rapid neutron capture process (r - process) for producing neutron-rich heavy elements. The capture rate of ν_e and $\bar{\nu}_e$ on neutrons and protons respectively determine the electron fraction, Y_e and for successful r-process Y_e must be less than 0.5. This is favored by the higher average energy of $\bar{\nu}_e$; however if oscillations between ν_e and ν_x takes place giving a stiffer ν_e spectrum, the r-process may get stopped. Thus to get r-process nucleosynthesis operative one excludes[53] the parameter space $\Delta m^2 >$ a few eV^2 and $\sin^2 2\theta < 10^{-5}$. Recently the effect of active-sterile neutrino transformation on the r-process was also considered[54] and initial work showed that it is possible to get sufficiently neutron rich matter to activate rapid neutron capture.

Acknowledgement

We thank G Dutta, D Indumathi, M V N Murthy and G Rajasekaran for allowing us to reproduce two of their figures.

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