Hadron energy resolution as a function of iron plate thickness and hadron direction resolution at INO ICAL

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January 10, 2013
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**Motivation**: As inputs for the determination of the energy and direction of atmospheric neutrinos interacting with iron nuclei in the ICAL detector via charged current (CC) and neutral current (NC) channels.

- **CC interactions**:
  \[ \nu_l + N \rightarrow l^- + X \]
  \[ \bar{\nu}_l + N \rightarrow l^+ + X \]

- **NC interactions**:
  \[ \nu_l + N \rightarrow \nu_l + X \]
  \[ \bar{\nu}_l + N \rightarrow \bar{\nu}_l + X \]

\[ l = e, \mu, \tau \]
\[ \nu_l = \text{neutrino of flavour } l \]
\[ N = \text{target nucleon} \]
\[ X = \text{hadronic final state} \]

**Figure 1**: Charged current and neutral current interactions of neutrino with nucleons.
ICAL can’t distinguish individual hadrons. Only a bunch of hits.
Majority: pions → Study for single pions.
No: of hadron hits depends on plate thickness & hadron energy.
11 different thicknesses from 1.5cm, ..., 8cm.
Gaussian fit over estimates the width of the distribution at low energies and higher thickness.

Figure 2: Hit distributions for 3 GeV and 8 GeV pions in (left) 6 cm and (right) 4 cm iron, fitted with Gaussian.
\[ \bar{n}(E) = n_0 \left[ 1 - \exp \left( -\frac{E}{E_0} \right) \right], \] where, \( n_0 \) and \( E_0 \) are constants.

\( E_0 \gg E \) in the range of energies of interest, \( E \leq 15 \text{GeV} \). Hence linearised by expanding the exponential: \( \bar{n}(E)/n_0 \approx E/E_0 \)

\[ \Delta n(E)/\bar{n}(E) = \sigma/E, \] where, \( \Delta n = \) width of the distribution, \( \bar{n}(E) = \) mean number of hits obtained from the distribution.

Parametrize \( \sigma(E)/E = \sqrt{a^2/E + b^2} \), where, \( a = \) stochastic coefficient (dependent on absorber thickness; has dimensions of \( \sqrt{E} \)), \( b = \) a dimensionless constant. Ideal case: \( b = 0 \).

\[ (\sigma/E)^2 = a^2/E + b^2 \] : easier to analyze since linear in \( 1/E \).

Analysis in \([2 \text{ GeV} - 5 \text{GeV}) \); \([5 \text{GeV} - 15 \text{GeV}] \); \([2 \text{GeV} - 15 \text{GeV}] \)

Thickness dependence: \( a(t) = p_0 t^{p_1} + p_2 \), where
\( p_0 = \) a constant,
\( p_1 = \) power giving the thickness dependence,
\( p_2 = \) residual resolution
<table>
<thead>
<tr>
<th>Plate thickness (t) (cm)</th>
<th>Stochastic coefficient (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.6</td>
</tr>
<tr>
<td>3</td>
<td>0.7</td>
</tr>
<tr>
<td>4</td>
<td>0.8</td>
</tr>
<tr>
<td>5</td>
<td>0.9</td>
</tr>
<tr>
<td>6</td>
<td>1.0</td>
</tr>
<tr>
<td>7</td>
<td>1.1</td>
</tr>
<tr>
<td>8</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Figure 3: $\alpha$ as a function of $t$ (cm) in various energy ranges. $p_0 = \text{constant}$, $p_1 = \text{exponent which gives the thickness dependence}$, $p_2 = \text{residual resolution}$
The exponent $p_1$ in the range $[5\text{GeV} - 15\text{GeV}] = 0.701 \pm 0.002 = p_0 t^{p_1} + p_2 = \alpha + p_2$

Constant term $p_2$ in this range $= 0.592 \pm 0.016 \rightarrow$ dominant residual resolution.

Optimisation: $\alpha$ improves with decreasing thickness, but very slightly. $\alpha_{5.6cm} = 0.276$

<table>
<thead>
<tr>
<th>$t$ (cm)</th>
<th>$\alpha_t$</th>
<th>$\Delta \alpha = \alpha_{5.6cm} - \alpha_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.221</td>
<td>0.055</td>
</tr>
<tr>
<td>2.5</td>
<td>0.162</td>
<td>0.114</td>
</tr>
<tr>
<td>1.5</td>
<td>0.167</td>
<td>0.167</td>
</tr>
</tbody>
</table>

5.6cm $\rightarrow$ say 2.5cm: increase in no:of layers, and hence no:of RPCs and other components.

Thickness $\downarrow$, cost $\uparrow$ and not much gain in resolution from hadron point of view.

Hence 4cm to 5.6cm are optimum thicknesses.
Hadron angle resolution

- For 5.6cm Fe only.
- Single, double pions at different $\theta$s with $\phi$ smeared fully; fixed $\theta$ - fixed $\phi$, hadrons from neutrino events.
- Direction reconstruction using hit information:
  1. **Centroid technique**
  2. **Orientation matrix method**
  3. **Raw hit method with timing**

**Centroid method**: for each simulated event, the vertex position and the positions of hits forming the shower are taken and the centroid of the shower is found by summing over the position vectors (w.r.t. the vertex) of each hit in that event → reconstructed shower direction.

**Orientation matrix method**: Orientation matrix $T$ for a collection of unit vectors $(x_i, y_i, z_i)$, $i=1,...,n$
Orientation matrix method...

\[ T = \begin{pmatrix} \sum x_i^2 & \sum x_iy_i & \sum x_iz_i \\ \sum x_iy_i & \sum y_i^2 & \sum y_iz_i \\ \sum x_iz_i & \sum y_iz_i & \sum z_i^2 \end{pmatrix} \]

Eigen analysis of this symmetric matrix \( \rightarrow \) idea of the shape of the underlying distribution. If a unit mass is placed at each point, moment of inertia of the \( n \) points about an arbitrary axis \((x_0, y_0, z_0)\) is,

\[ n - (x_0 \ y_0 \ z_0) T \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} \]

The variation of moment of inertia gives information about the scatter of the points as the choice of axis varies. The axis about which the moment is least \( \rightarrow \) principal axis \( \rightarrow \) shower direction.

Distributions of the sine of the error angles \((\sin \Delta \theta)\) fitted with the function : \( \Delta \theta = A \Delta \theta \exp(-B \Delta \theta) \), where, \( A \) and \( B \) parameters.
Figure 4: $\Delta \theta$ distribution obtained using the two techniques at 2GeV, 3Gev, 6GeV and 10GeV (clockwise from top-left).
Raw hits and timing method

- No vertex position is needed. Only hit information in X-Z and Y-Z plane separately. Time window of ≤50 ns.
- Average $x$ and $y$ positions in the $i^{th}$ layer of an event are found separately.
- Fitted with straight lines $x = m'_xz + c_1$ and $y = m'_yz + c_2$ separately in the X-Z and Y-Z planes. Inverses of slopes $m'_x$ and $m'_y \rightarrow$ reconstruct the direction. $m_x$ and $m_y$.
- Using polar co-ordinates, $\theta$ and $\phi$ can be reconstructed as:
  $$\tan \phi = \frac{\tan \omega}{\tan \lambda} \& \tan \theta = \frac{1}{\cot \theta}$$, where, $\omega =$ angle made by a line with the X axis, in the XZ plane and $\lambda =$ angle made by a line with the Y axis in the YZ plane.
- Timing information $\rightarrow$ to break the quadrant degeneracy of $m_x$ and $m_y$.
  - All events UP in time $\rightarrow \theta$ in $1^{st}$ quadrant.
  - All events DOWN in time $\rightarrow \theta$ in $2^{nd}$ quadrant.
Figure 5: $\theta$ resolution in degrees for (a) single pions and (b) double pions with $l_{\text{min}} = 2$ cut
Single and double pions in a fixed direction (fixed $\theta$ - fixed $\phi$) :

Figure 6: Comparison of $\theta$ resolution at 30$^\circ$ for (left) single pions and (right) double pions propagated in the fixed direction $\theta = 30^\circ$ and $\phi = 30^\circ$ with $l_{\text{min}} = 2$ cut.
• Hadrons are reconstructed as showers in the detector since they don’t leave clean tracks like muons to which ICAL is most sensitive.

• Even then it is possible to extract their direction information from hit pattern.

• Resolution worsens in the realistic case of several hadrons in the final state since multiple hadrons may travel in different directions thus giving hits in a larger region (larger spread).
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5. K. Kleinknecht, *Particle Detectors*, Institut für Physik der Universität Dortmund, Dortmund, Germany
Acknowledgements

Heartfelt thanks to Prof. Amol Dighe, TIFR, Mumbai and Prof. D. Indumathi, IMSc, Chennai. Also to Prof. M.V.N. Murthy, IMSc, Chennai, Prof. N. K. Mondal, TIFR, Mumbai, Prof. Gobinda Majumdar, TIFR, Mumbai and Asmita Redij. THANK YOU